

Some Deep Learning for Neuroscientists

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MAIN 2019

(Slides ruthlessly stolen from Andrej Karpathy)

Today's Goals:

Theoretical

- Understand when to bother using deep learning.
- Understand the basic math behind training a deep learning neural network.

Practical (Optional)

- See how this is done with a simple feedforward network in **Numpy**.

What's the deal with
deep learning?

Image Classification: a core task in Computer Vision



(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

→ cat

The problem: *semantic gap*

Images are represented as
3D arrays of numbers, with
integers between [0, 255].

E.g.
300 x 100 x 3

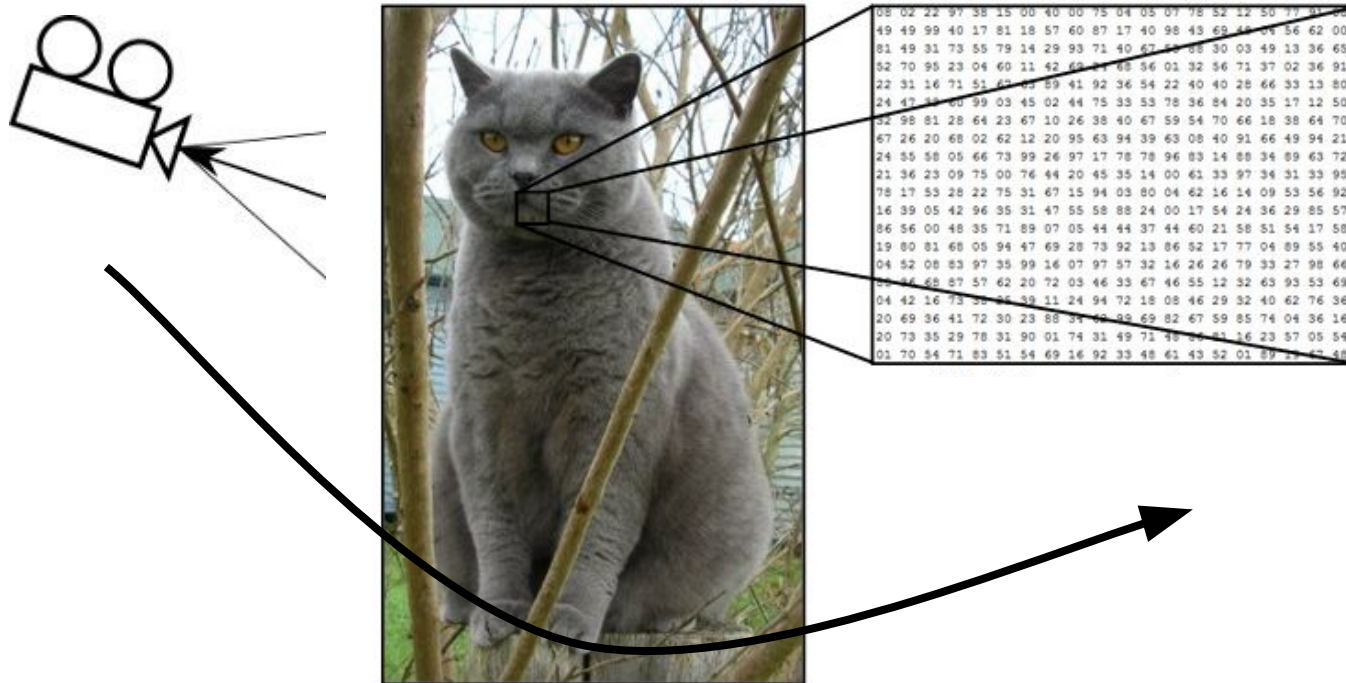
(3 for 3 color channels RGB)



08	02	22	97	38	15	00	40	00	75	04	05	07	78	52	12	50	77	91	71
49	49	99	40	17	81	18	57	60	87	17	40	98	43	69	48	04	56	62	00
81	49	31	73	55	79	14	29	93	71	40	67	53	88	30	03	49	13	36	65
52	70	98	23	04	60	11	42	60	71	68	56	01	32	56	71	37	02	36	91
22	31	16	71	51	67	03	89	41	92	36	54	22	40	40	28	66	33	13	80
24	47	38	60	99	03	45	02	44	75	33	53	78	36	84	20	35	17	12	58
32	98	81	28	64	23	67	10	26	38	40	67	59	54	70	66	18	38	64	70
67	26	20	68	02	62	12	20	95	63	94	39	63	08	40	91	66	49	94	21
24	55	58	05	66	73	99	26	97	17	78	78	96	83	14	88	34	89	63	72
21	36	23	09	75	00	76	44	20	45	35	14	00	61	33	97	34	31	33	95
78	17	53	28	22	75	31	67	15	94	03	80	04	62	16	14	09	53	56	92
16	39	05	42	96	35	31	47	55	58	88	24	00	17	54	24	36	29	85	57
86	56	00	48	35	71	89	07	05	44	44	37	44	60	21	58	51	54	17	58
19	80	81	68	05	94	47	69	28	73	92	13	86	52	17	77	04	89	55	40
04	52	08	83	97	35	99	16	07	97	57	32	16	26	26	79	33	27	98	66
85	57	68	87	57	62	20	72	03	46	33	67	46	55	12	32	63	93	53	69
04	42	16	73	58	37	39	11	24	94	72	18	08	46	29	32	40	62	76	36
20	69	36	41	72	30	23	88	31	72	89	69	82	67	59	85	74	04	36	16
20	73	35	29	78	31	90	01	74	31	49	71	48	88	81	16	23	57	05	54
01	70	54	71	83	51	54	69	16	92	33	48	61	43	52	01	89	51	57	48

What the computer sees

Challenges: Viewpoint Variation



Challenges: Illumination



Challenges: Deformation



Challenges: Occlusion



Challenges: Background clutter



Challenges: Intraclass variation



An image classifier

```
def predict(image):  
    # ????  
    return class_label
```

Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for recognizing a cat, or other classes.

How do we compare the images? What is the **distance metric**?

L1 distance:
$$d_1(I_1, I_2) = \sum_P |I_1^P - I_2^P|$$

test image		training image		pixel-wise absolute value differences			
56	32	10	18	46	12	14	1
90	23	128	133	82	13	39	33
24	26	178	200	12	10	0	30
2	0	255	220	2	32	22	108

add → 456

Nearest Neighbor classifier

```
import numpy as np

class NearestNeighbor:
    def __init__(self):
        pass

    def train(self, X, y):
        """ X is N x D where each row is an example. Y is 1-dimension of size N """
        # the nearest neighbor classifier simply remembers all the training data
        self.Xtr = X
        self.ytr = y

    def predict(self, X):
        """ X is N x D where each row is an example we wish to predict label for """
        num_test = X.shape[0]
        # lets make sure that the output type matches the input type
        Ypred = np.zeros(num_test, dtype = self.ytr.dtype)

        # loop over all test rows
        for i in xrange(num_test):
            # find the nearest training image to the i'th test image
            # using the L1 distance (sum of absolute value differences)
            distances = np.sum(np.abs(self.Xtr - X[i,:]), axis = 1)
            min_index = np.argmin(distances) # get the index with smallest distance
            Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

        return Ypred
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Nearest Neighbor classifier

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import numpy as np
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```

remember the training data

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        return Ypred
```

- for every test image:
- find nearest train image with L1 distance
 - predict the label of nearest training image


```
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```

```
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```

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Nearest Neighbor classifier

Q: how does the classification speed depend on the size of the training data?

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Nearest Neighbor classifier

Q: how does the classification speed depend on the size of the training data?
linearly :(

This is **backwards**:

- test time performance is usually much more important in practice.
- CNNs flip this: expensive training, cheap test evaluation

The choice of distance is a **hyperparameter**
common choices:

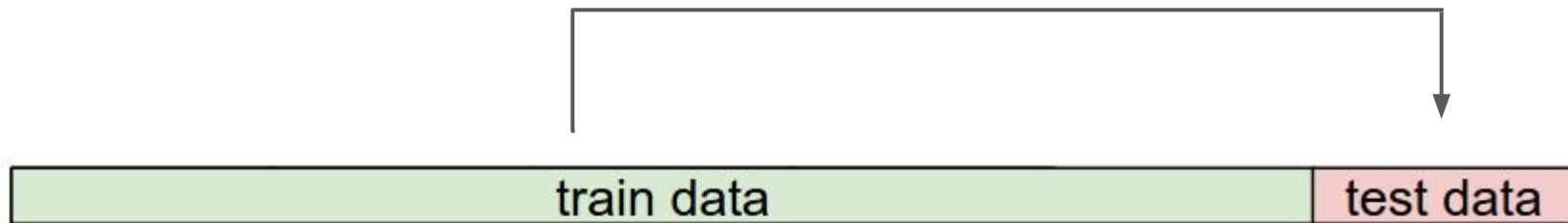
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$

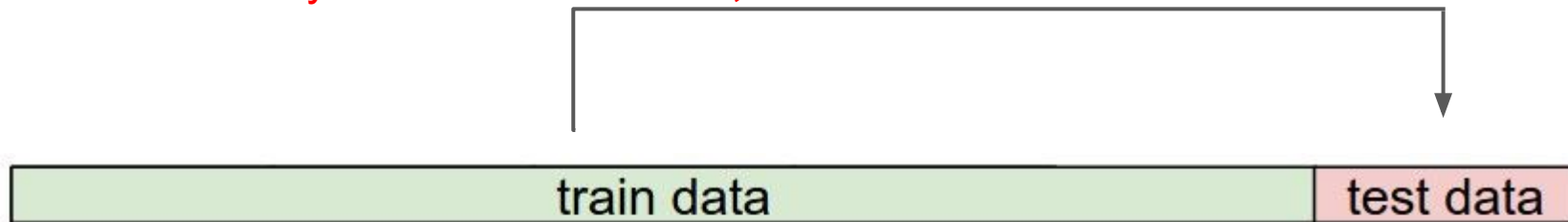
Try out what hyperparameters work best on test set.

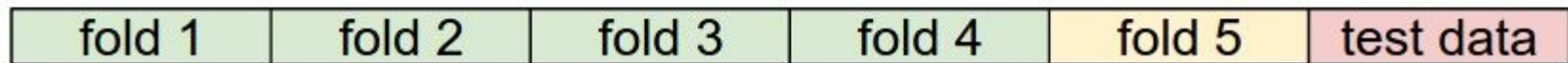
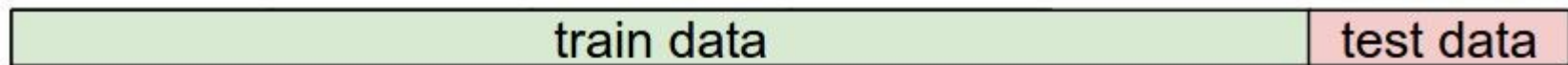


Trying out what hyperparameters work best on test set:

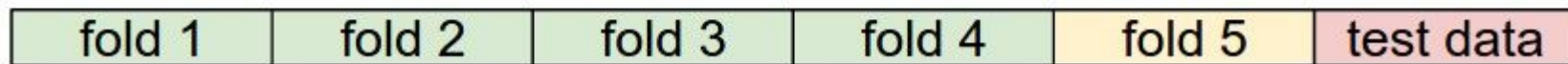
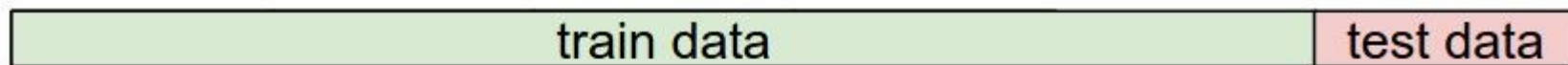
Very bad idea. The test set is a proxy for the generalization performance!

Use only **VERY SPARINGLY**, at the end.





Validation data
use to tune hyperparameters

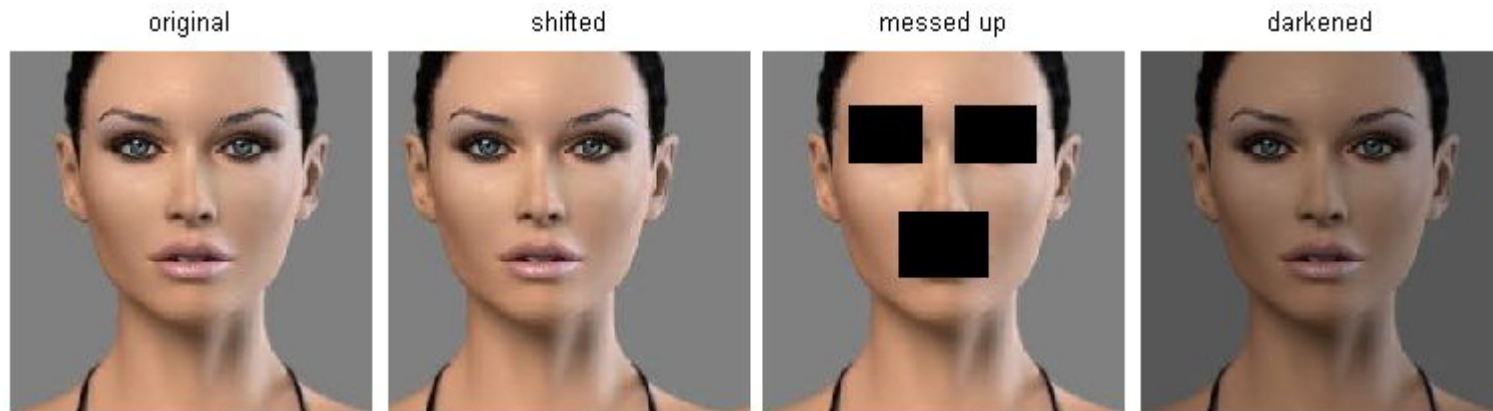


Cross-validation

cycle through the choice of which fold is the validation fold, average results.

k-Nearest Neighbor on images **never used**.

- terrible performance at test time
- distance metrics on level of whole images can be very unintuitive

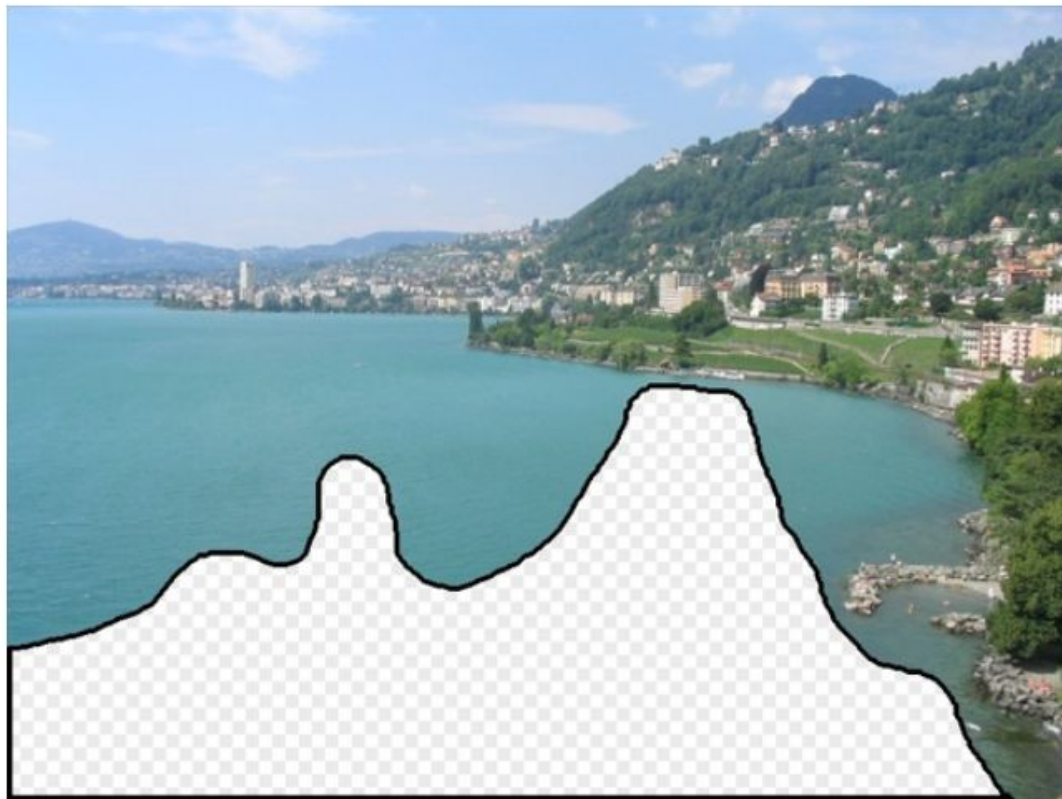


(all 3 images have same L2 distance to the one on the left)

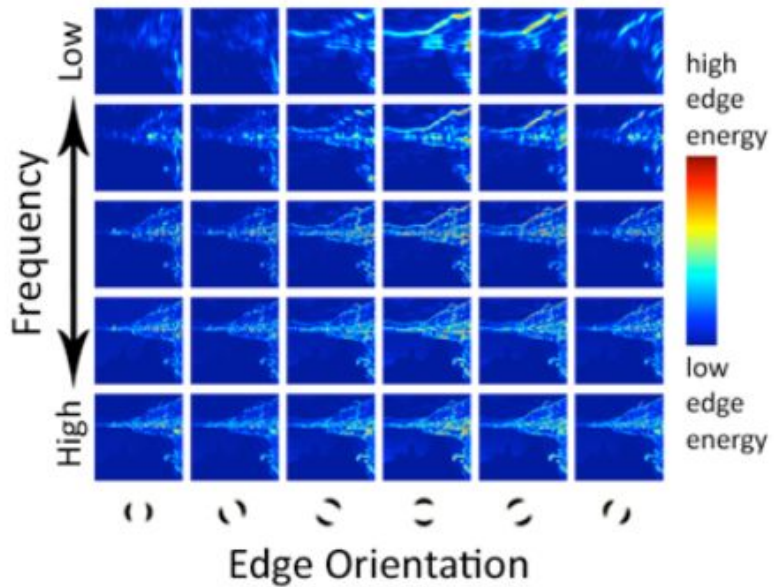
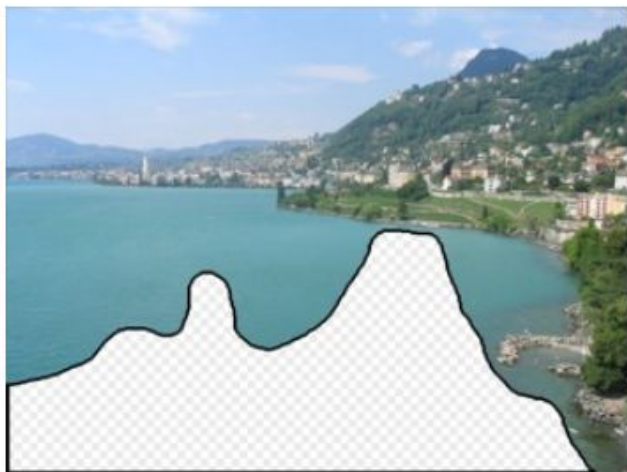
Scene Completion



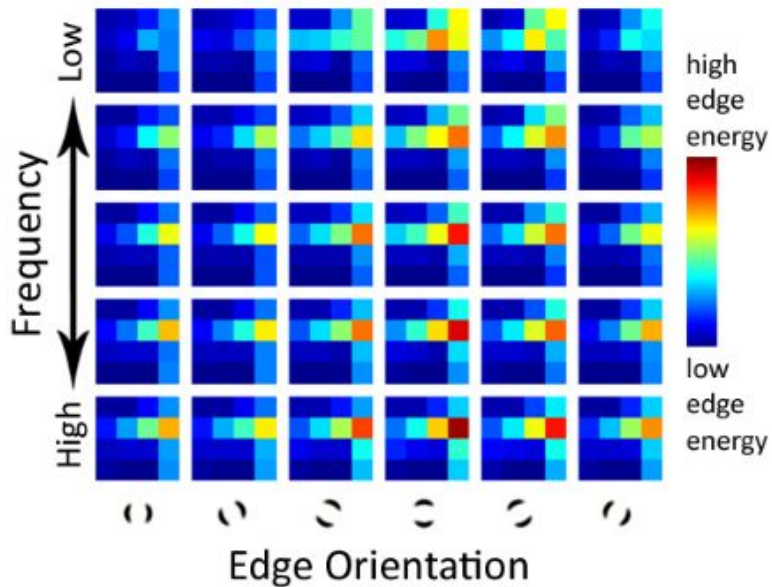
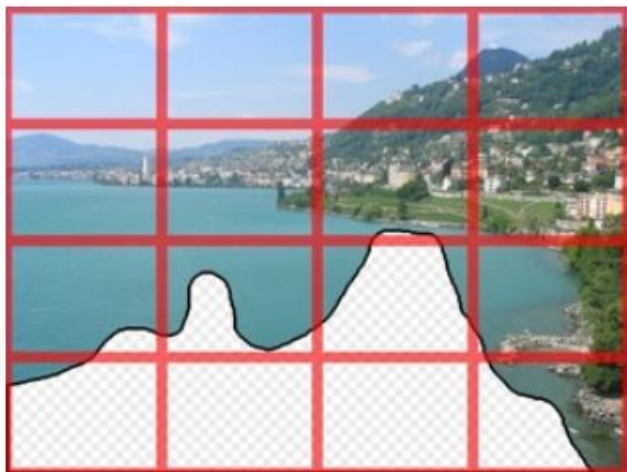
Scene Matching



Scene Descriptor

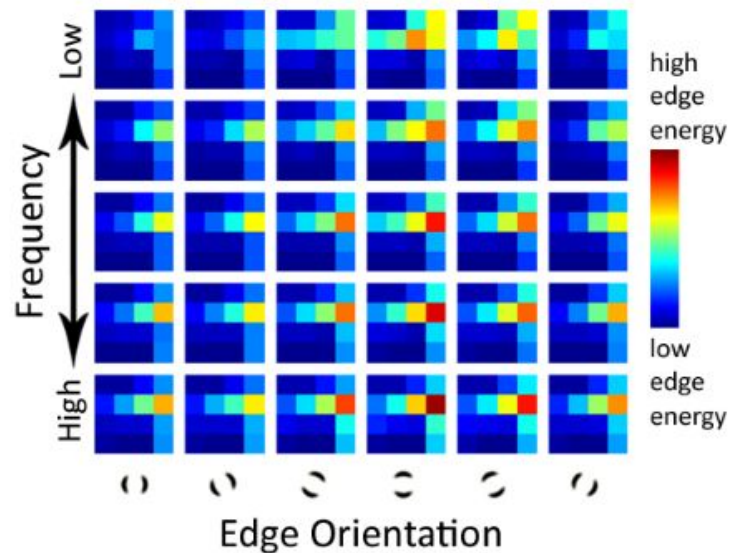
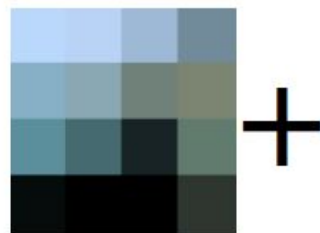


Scene Descriptor



Scene Gist Descriptor
(Oliva and Torralba 2001)

Scene Descriptor

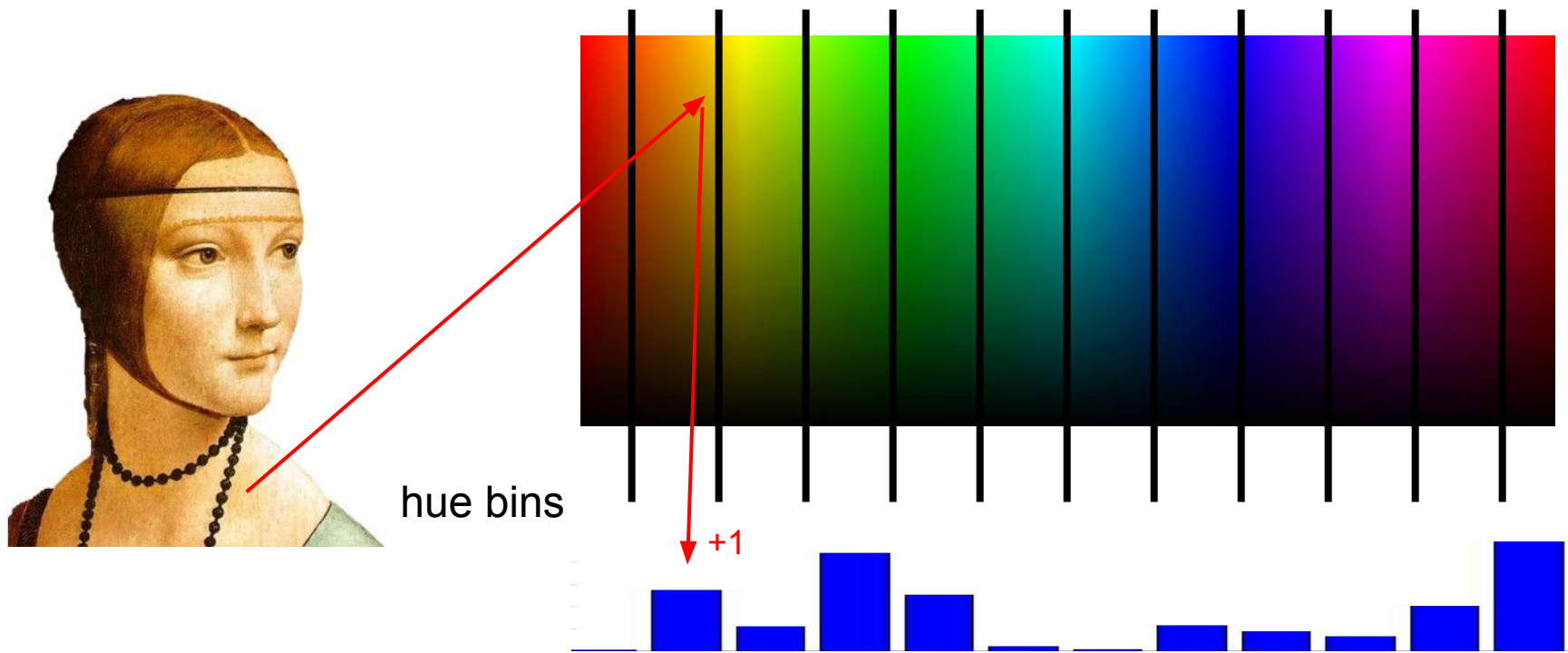


Scene Gist Descriptor
(Oliva and Torralba 2001)



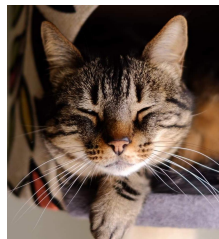
2 Million Flickr Images → 200 matches.

Example: Color (Hue) Histogram

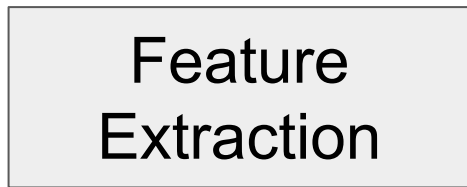


Let's be lazy instead

- Instead of trying to enumerate all of the possible functions requires to decompose images, let's just use neural networks which can **learn on their own** what those functions are!
- Neural networks are “universal function approximators”.
<http://neuralnetworksanddeeplearning.com/chap4.html>



[32x32x3]



vector describing various
image statistics



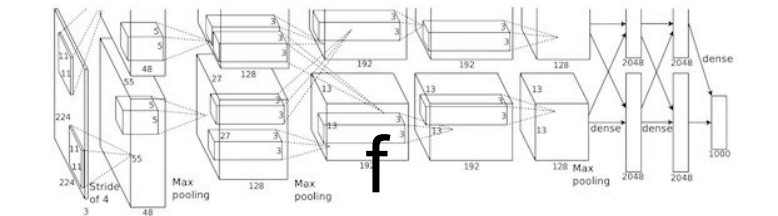
f

← training

10 numbers, indicating
class scores



[32x32x3]



← training

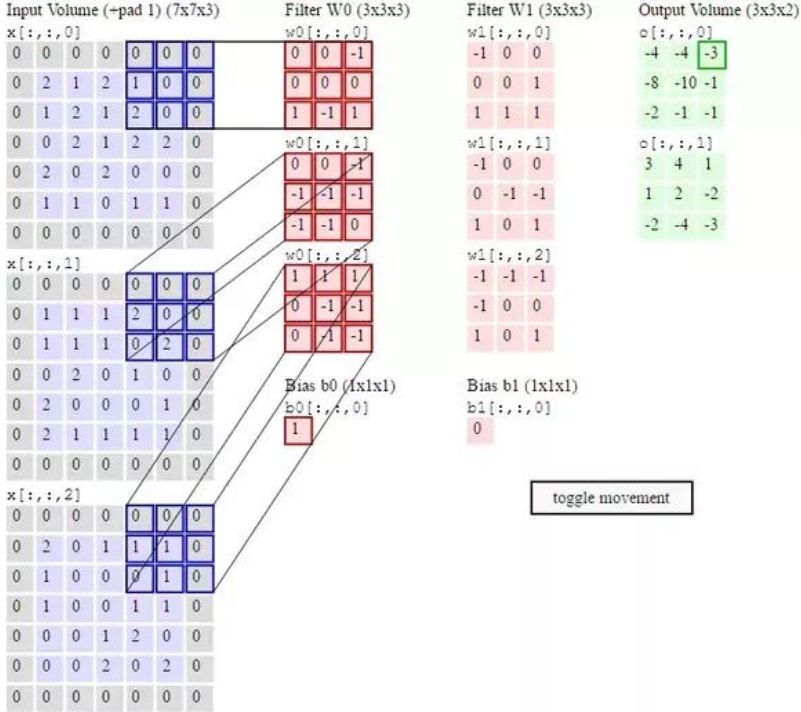
10 numbers, indicating
class scores

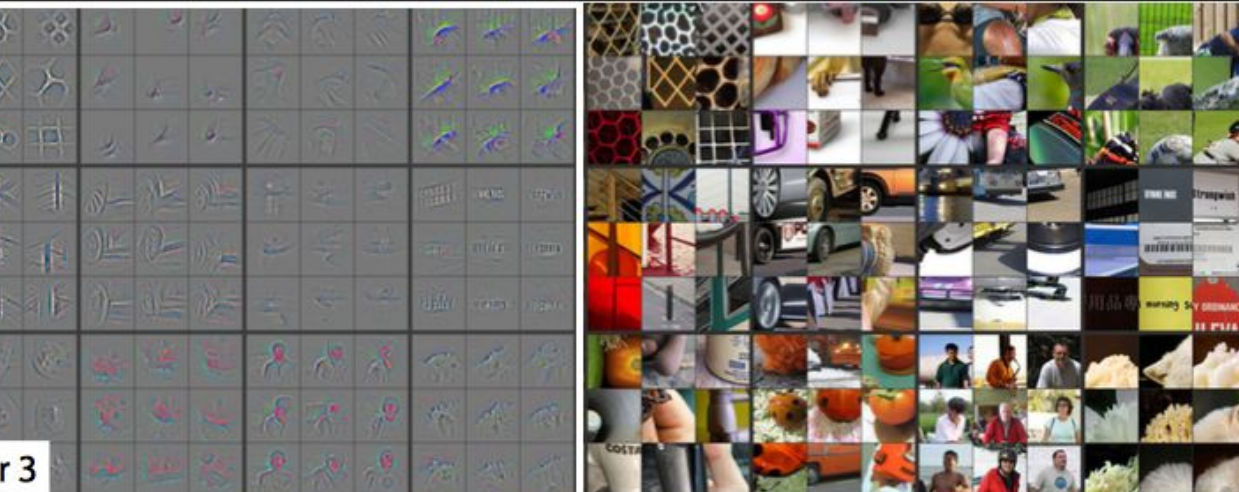
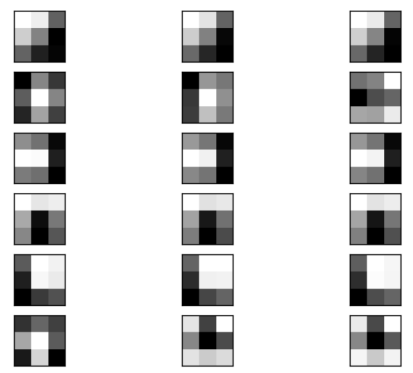
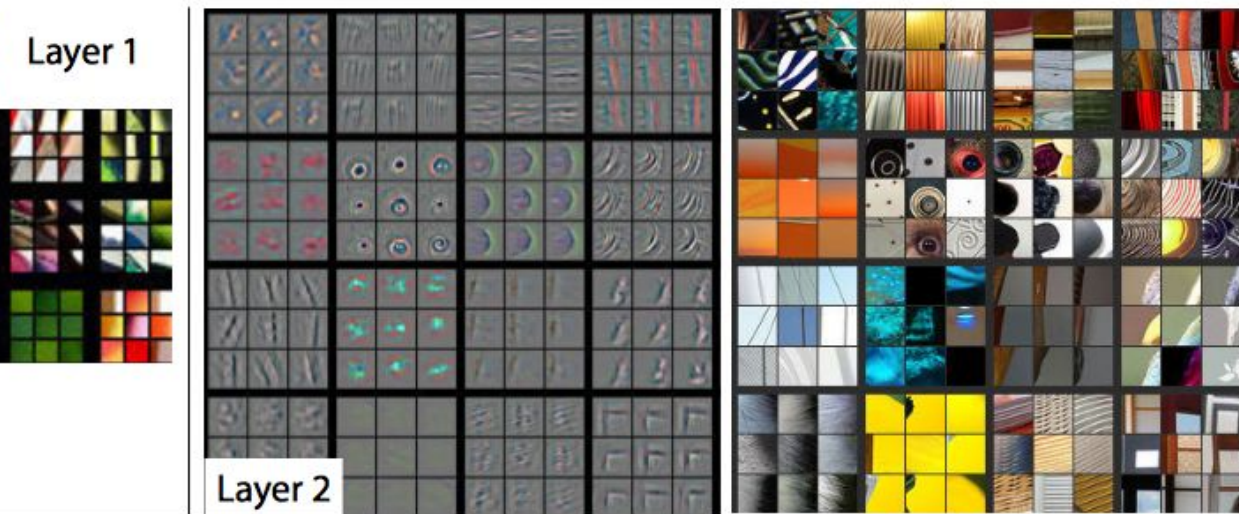


Enter Convolutional Neural Networks

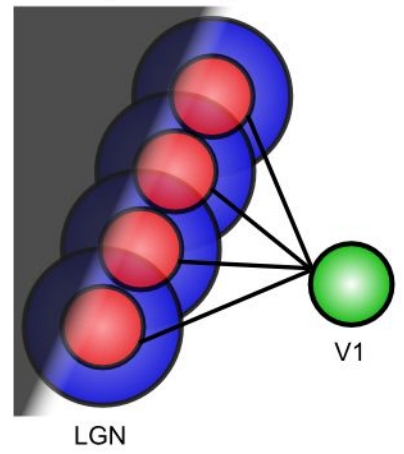
ONLY ASSUMPTION: Things occur close together in the inputs (like in images)!

Hyperparameter: Kernel size!





V1 simple cell edge detector



Why don't we use neural networks for everything?

- Neural networks are **too strong**: if you let them, they will just memorize the training data and not work on any new data.
 - **Regularization.**
 - **Lots of training data (the best regularizer).**
 - Specific model architectures.

When should I consider a neural network?

- Neural networks are a **good** candidate when we have lots of:
 - Data.
 - Time (your time).
 - No idea what the functions generating the data might be.

How to know if neural networks are appropriate?

- Establish a **baseline!**
 - **Do it now!**
- You will be surprised how well a linear regression / SVM / random forest model (3 lines of code in scikit-learn) will perform on your task if you have good feature engineering done already.

Summary

- Deep learning does (some) feature engineering for us.
- Deep learning is slow to train but quick to predict!
- Still required to pick **hyperparameters** and evaluate those choices on **held-out data**.
- Otherwise we will **overfit** and our model will be useless.
 - **Generalize** don't **Memorize**.

How does deep
learning work?

Parametric approach

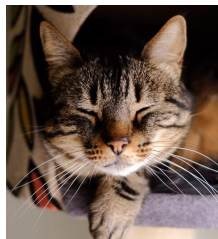


image parameters

$$f(\mathbf{x}, \mathbf{W})$$



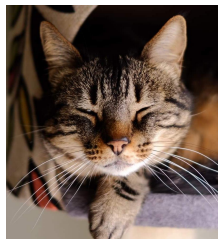
10 numbers,
indicating class
scores

[32x32x3]

array of numbers 0...1
(3072 numbers total)

Parametric approach: **Linear classifier**

$$f(x, W) = Wx$$

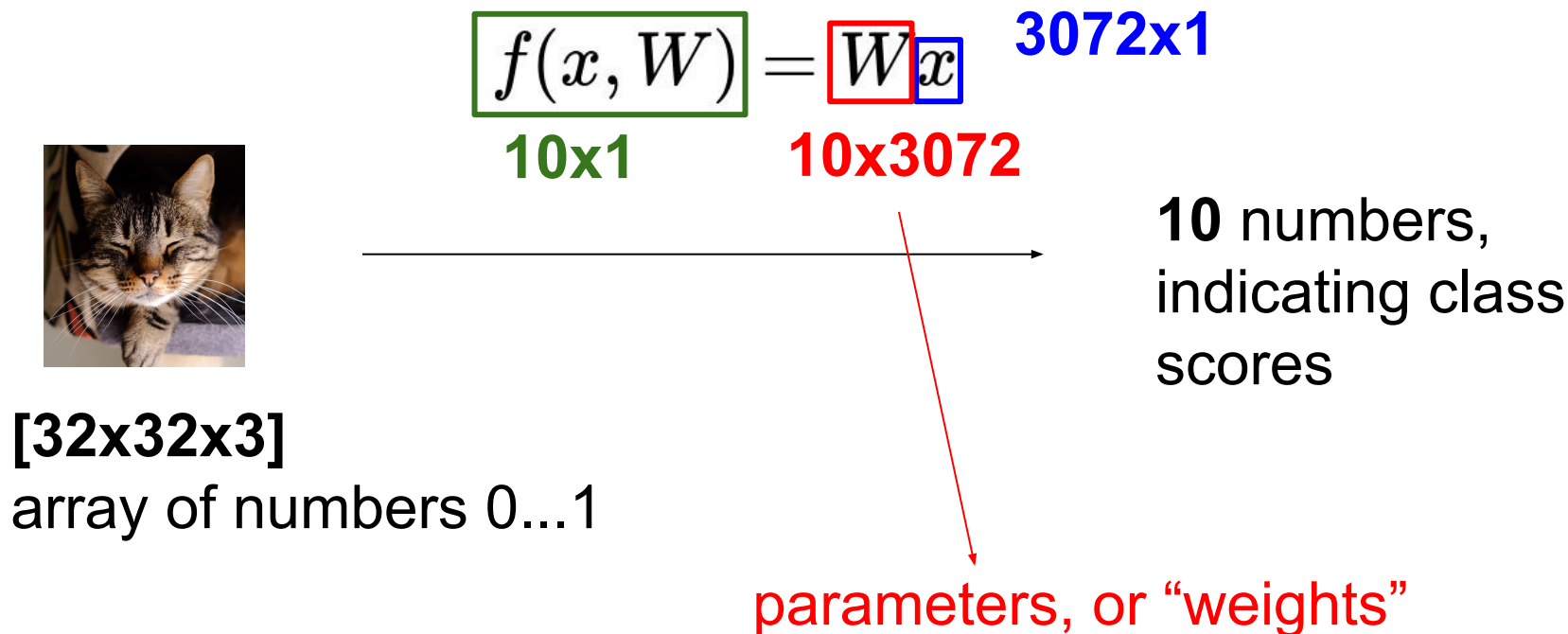


10 numbers,
indicating class
scores

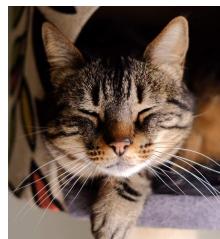
[32x32x3]

array of numbers 0...1

Parametric approach: Linear classifier



Parametric approach: Linear classifier

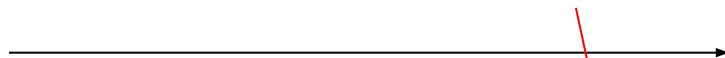


[32x32x3]

array of numbers 0...1

$$f(x, W) = Wx \quad (+b)$$

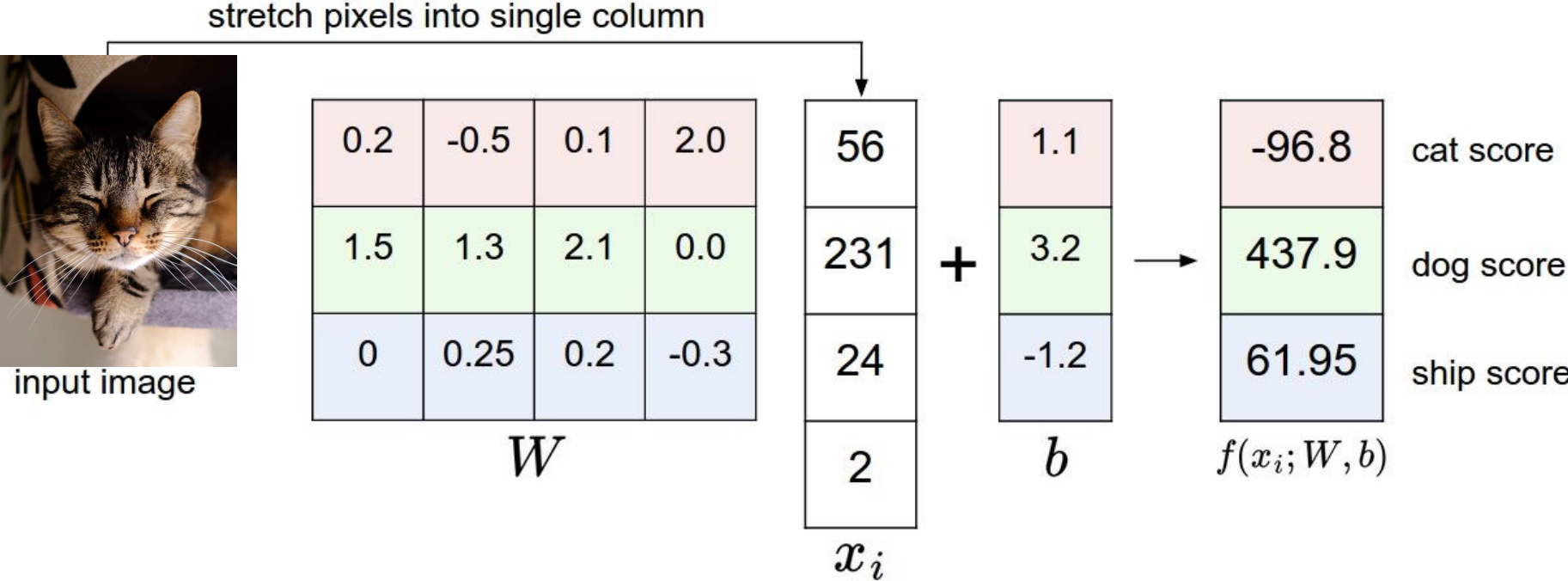
10x1 **10x3072** **3072x1** **10x1**



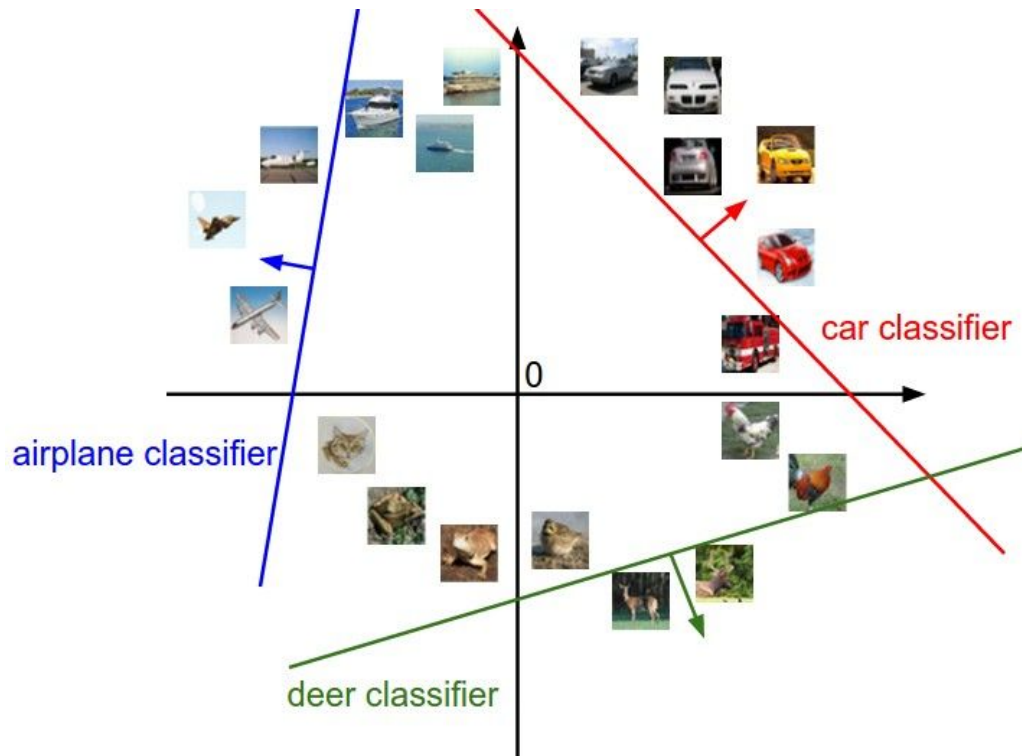
10 numbers,
indicating class
scores

parameters, or “weights”

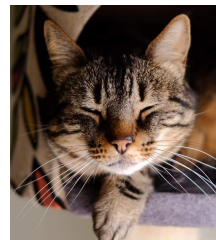
Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Interpreting a Linear Classifier



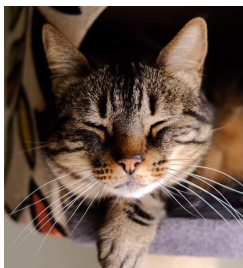
$$f(x_i, W, b) = Wx_i + b$$



[32x32x3]
array of numbers 0...1
(3072 numbers total)

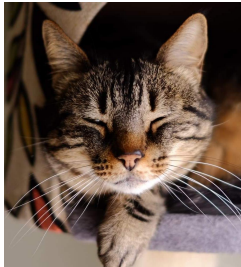
Suppose: 3 training examples, 3 classes.

With some W the scores $f(x, W) = Wx$ are:



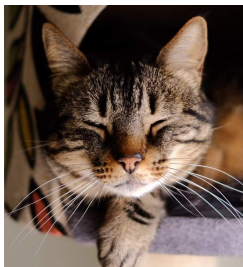
cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1

Softmax Classifier (Multinomial Logistic Regression)



cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

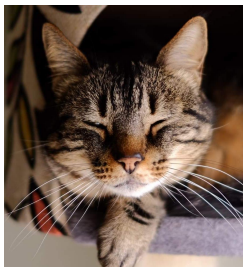


scores = unnormalized log probabilities of the classes.

$$s = f(x_i; W)$$

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

where

$$s = f(x_i; W)$$

cat	3.2
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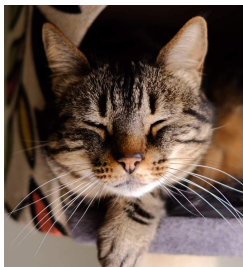
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Softmax function

Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

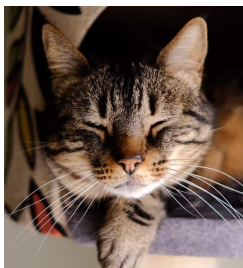
$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \quad \text{where} \quad s = f(x_i; W)$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i | X = x_i)$$

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \quad \text{where} \quad s = f(x_i; W)$$

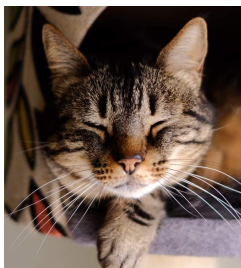
Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i|X = x_i)$$

cat	3.2
car	5.1
frog	-1.7

in summary: $L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$

Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

cat

3.2

car

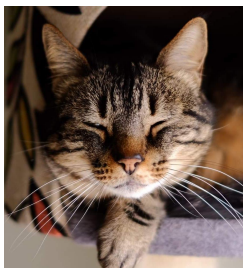
5.1

frog

-1.7

unnormalized log probabilities

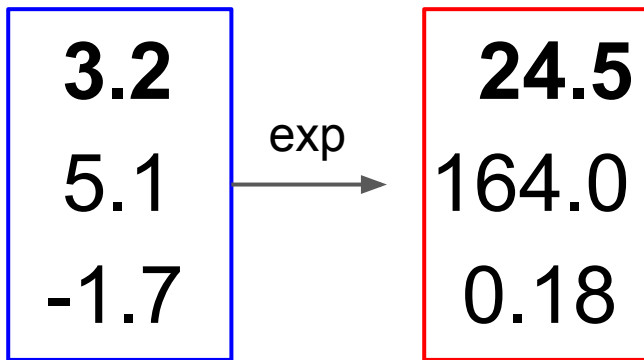
Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat
car
frog



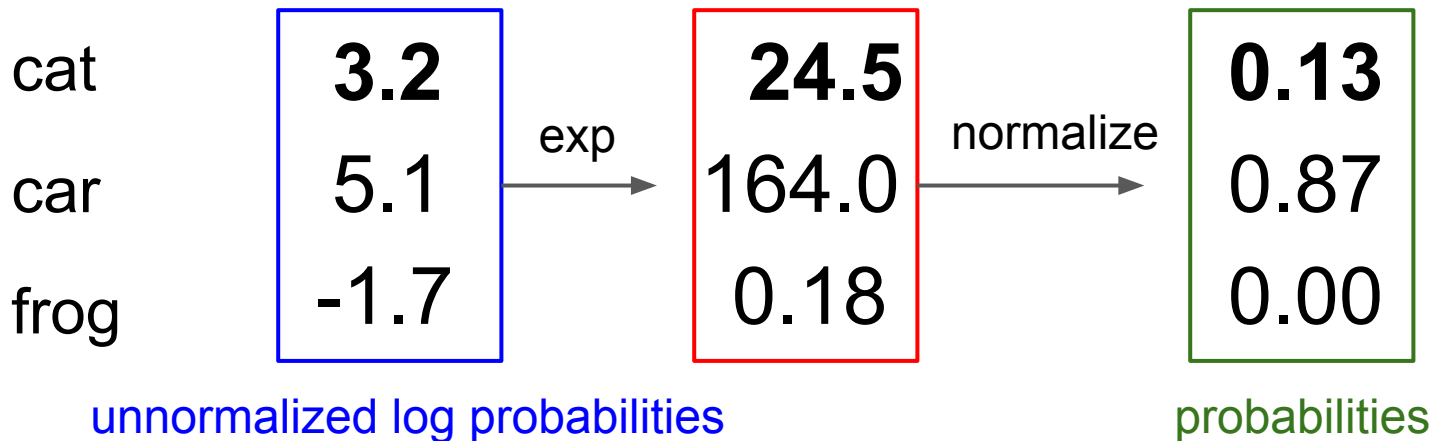
unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)

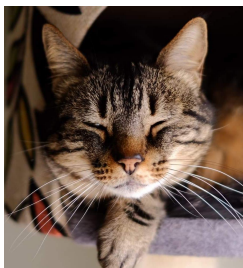


$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities



Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat
car
frog

3.2
5.1
-1.7

exp

24.5
164.0
0.18

normalize

0.13
0.87
0.00

$$L_i = -\log(0.13) = 0.89$$

unnormalized log probabilities

probabilities

Goal: optimize the weights \mathbf{W} to minimize the loss L .

$$= \arg \min_{\boldsymbol{\theta}} \frac{1}{n} \sum_{i=1}^n L(y^{(i)}, f(\mathbf{x}^{(i)}, \boldsymbol{\theta}))$$

Strategy #1: A first very bad idea solution: **Random search**

```
# assume X_train is the data where each column is an example (e.g. 3073 x 50,000)
# assume Y_train are the labels (e.g. 1D array of 50,000)
# assume the function L evaluates the loss function

bestloss = float("inf") # Python assigns the highest possible float value
for num in xrange(1000):
    W = np.random.randn(10, 3073) * 0.0001 # generate random parameters
    loss = L(X_train, Y_train, W) # get the loss over the entire training set
    if loss < bestloss: # keep track of the best solution
        bestloss = loss
        bestW = W
    print 'in attempt %d the loss was %f, best %f' % (num, loss, bestloss)

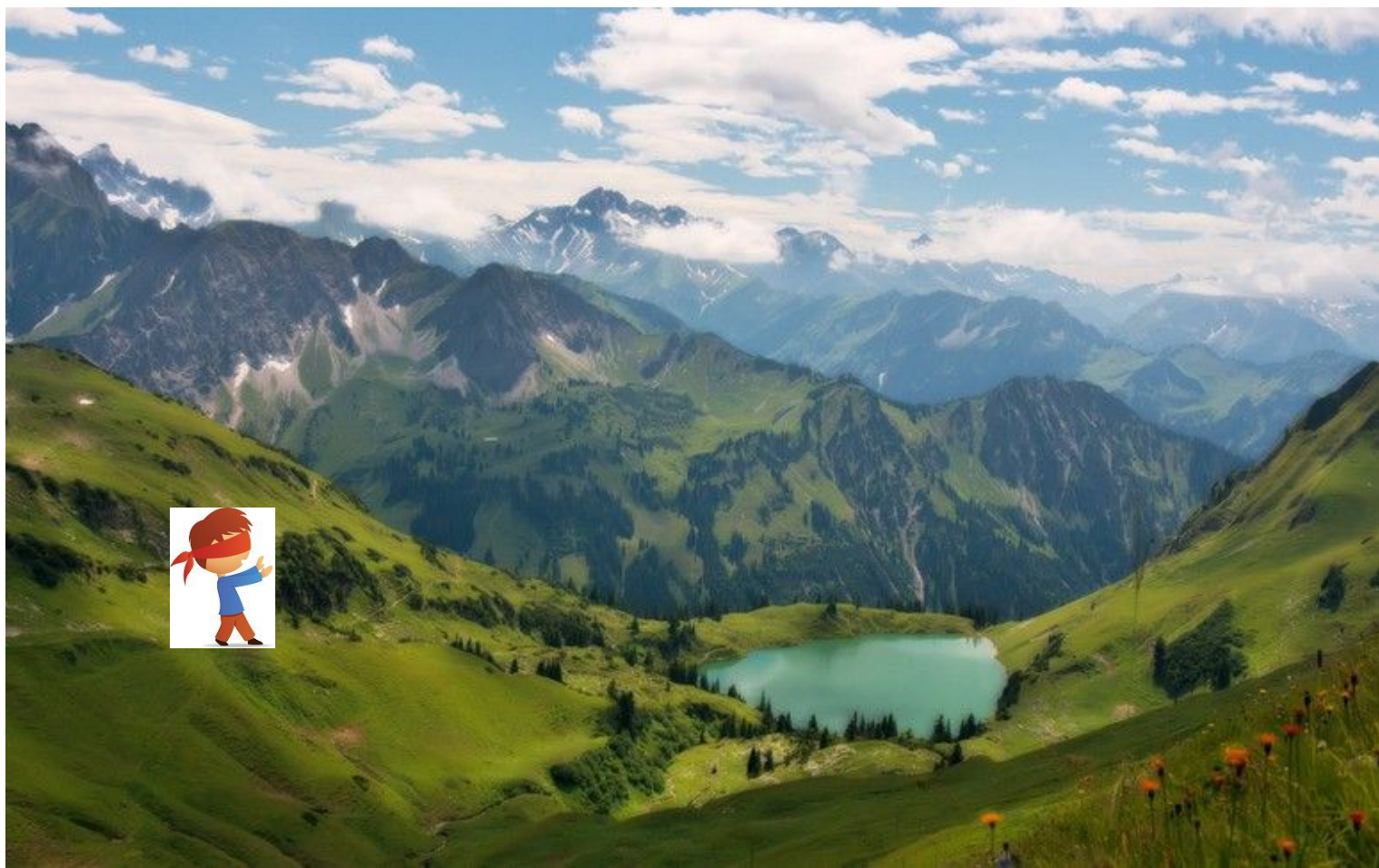
# prints:
# in attempt 0 the loss was 9.401632, best 9.401632
# in attempt 1 the loss was 8.959668, best 8.959668
# in attempt 2 the loss was 9.044034, best 8.959668
# in attempt 3 the loss was 9.278948, best 8.959668
# in attempt 4 the loss was 8.857370, best 8.857370
# in attempt 5 the loss was 8.943151, best 8.857370
# in attempt 6 the loss was 8.605604, best 8.605604
# ... (truncated: continues for 1000 lines)
```

Lets see how well this works on the test set...

```
# Assume X_test is [3073 x 10000], Y_test [10000 x 1]  
scores = Wbest.dot(Xte_cols) # 10 x 10000, the class scores for all test examples  
# find the index with max score in each column (the predicted class)  
Yte_predict = np.argmax(scores, axis = 0)  
# and calculate accuracy (fraction of predictions that are correct)  
np.mean(Yte_predict == Yte)  
# returns 0.1555
```

15.5% accuracy! not bad!
(SOTA is ~95%)





Strategy #2: Follow the slope

In 1-dimension, the derivative of a function:

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

In multiple dimensions, the **gradient** is the vector of (partial derivatives).

The loss is just a function of W , so

$$s = f(x; W) = Wx$$

want $\nabla_W L$

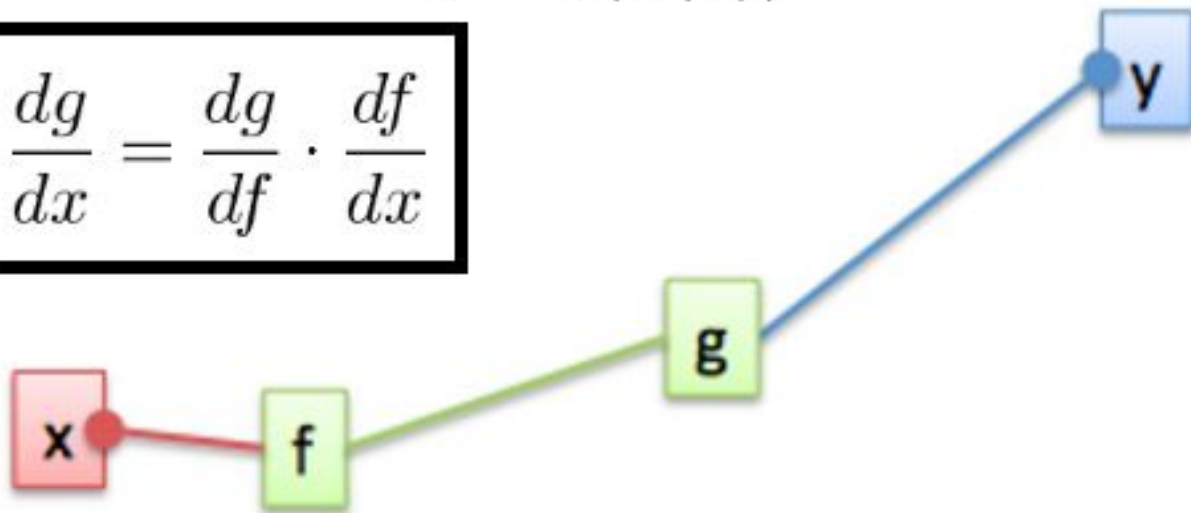
Calculus



Chain Rule

$$y = g(f(x))$$

$$\frac{dg}{dx} = \frac{dg}{df} \cdot \frac{df}{dx}$$



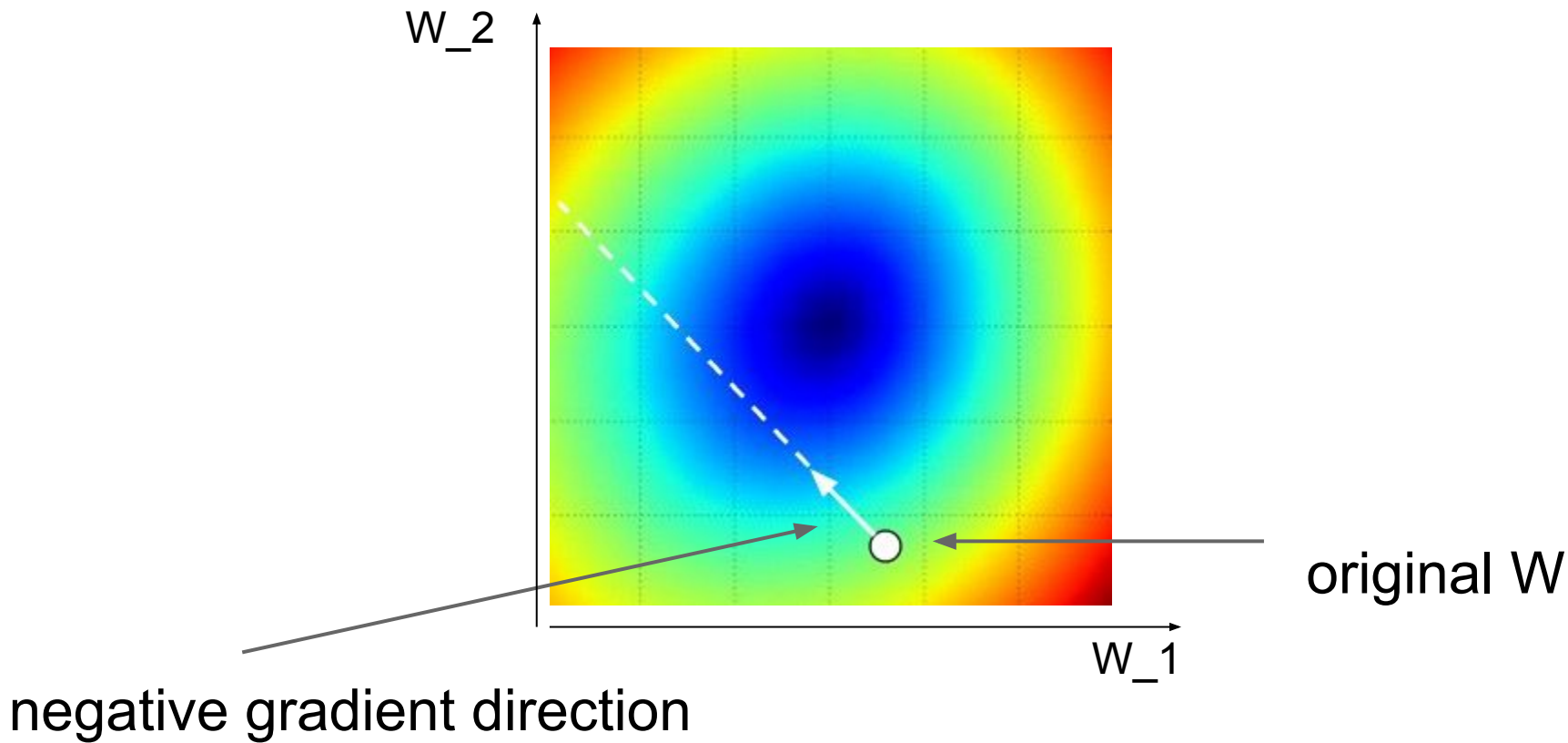
Gradient Descent

```
# Vanilla Gradient Descent
```

```
while True:
```

```
    weights_grad = evaluate_gradient(loss_fun, data, weights)
```

```
    weights += - step_size * weights_grad # perform parameter update
```

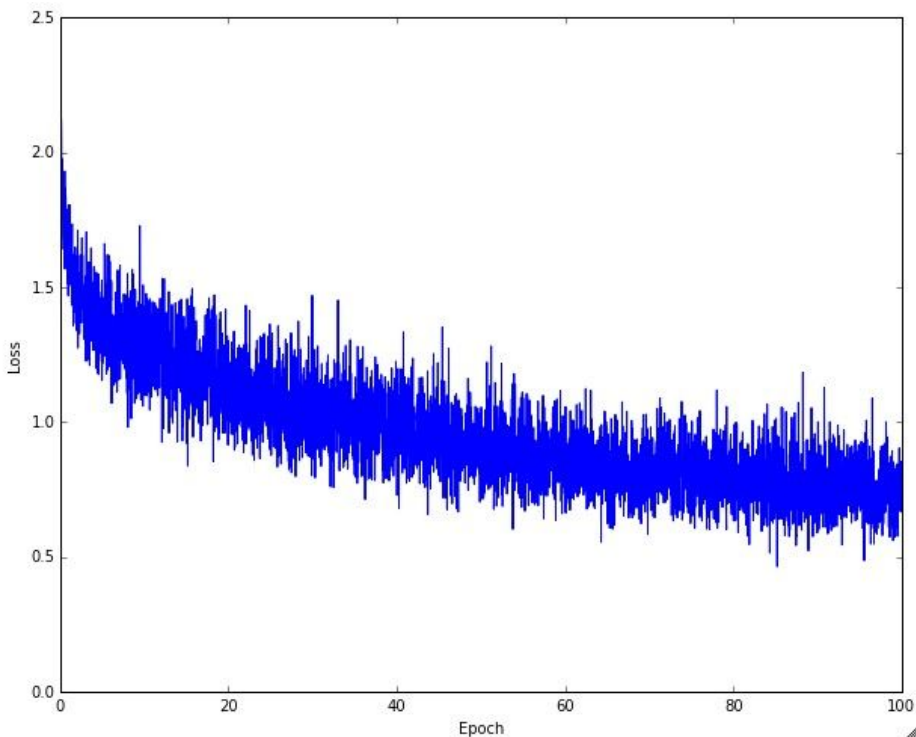


Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

```
# Vanilla Minibatch Gradient Descent  
  
while True:  
    data_batch = sample_training_data(data, 256) # sample 256 examples  
    weights_grad = evaluate_gradient(loss_fun, data_batch, weights)  
    weights += - step_size * weights_grad # perform parameter update
```

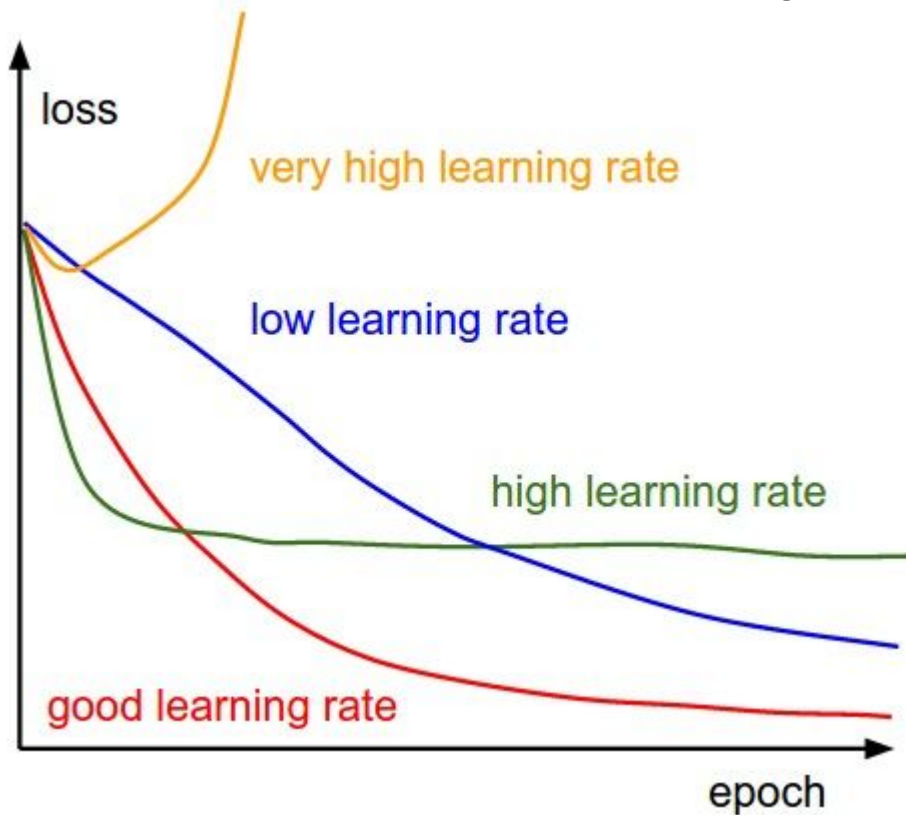
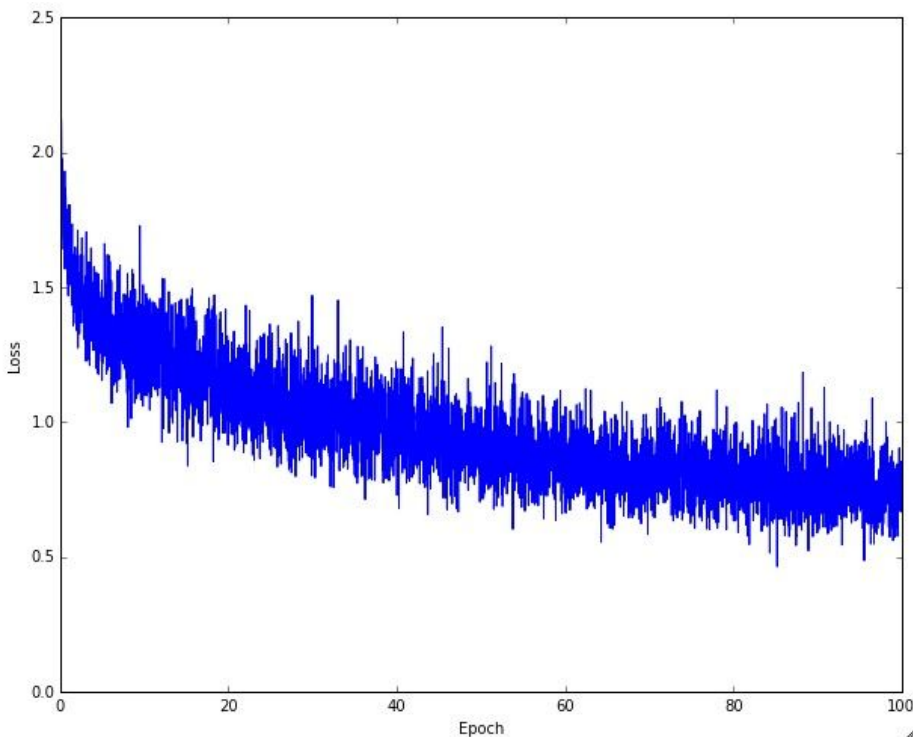
Common mini-batch sizes are 32/64/128 examples
e.g. Krizhevsky ILSVRC ConvNet used 256 examples



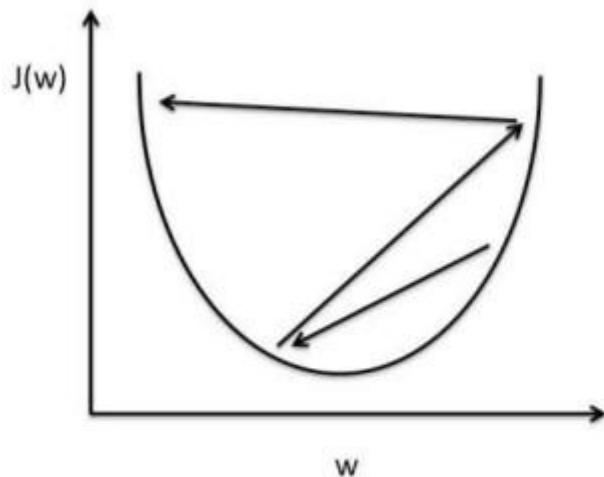
Example of optimization progress while training a neural network.

(Loss over mini-batches goes down over time.)

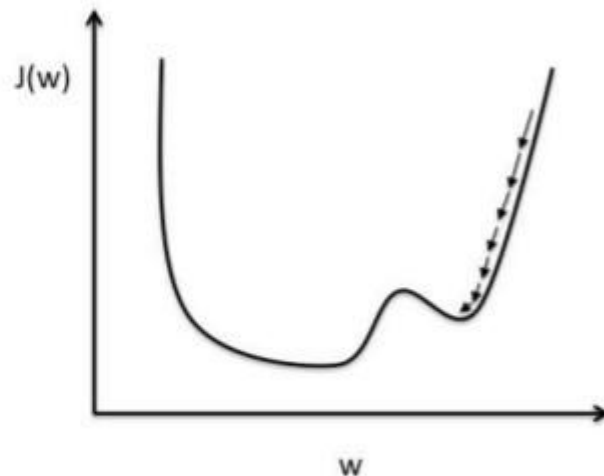
The effects of step size (or “learning rate”)



Learning rate



Overshooting



Learn too slow

Neural Network: without the brain stuff

(**Before**) Linear score function: $f = Wx$

Neural Network: without the brain stuff

(**Before**) Linear score function:

$$f = Wx$$

(**Now**) 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$

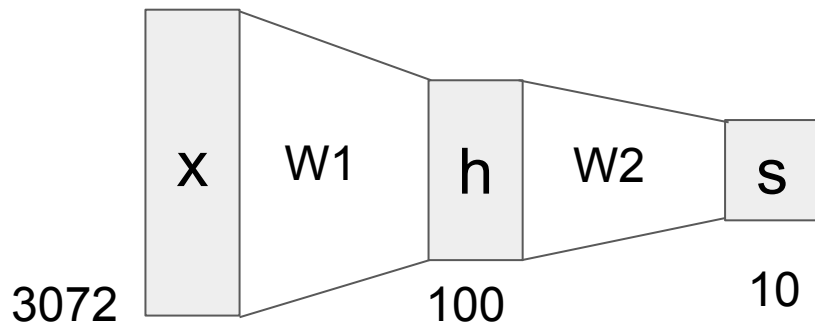
Neural Network: without the brain stuff

(**Before**) Linear score function:

$$f = Wx$$

(**Now**) 2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$



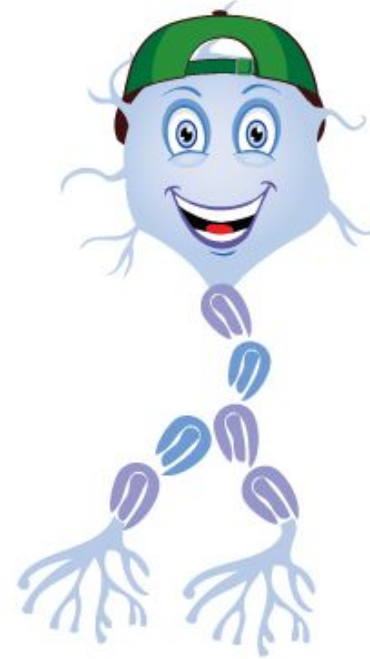
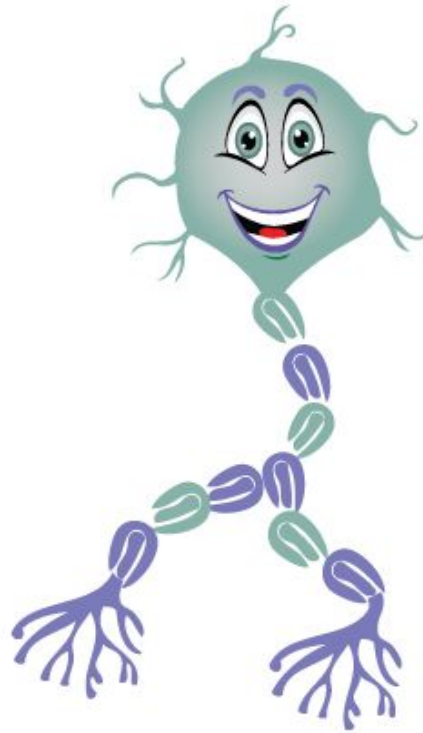
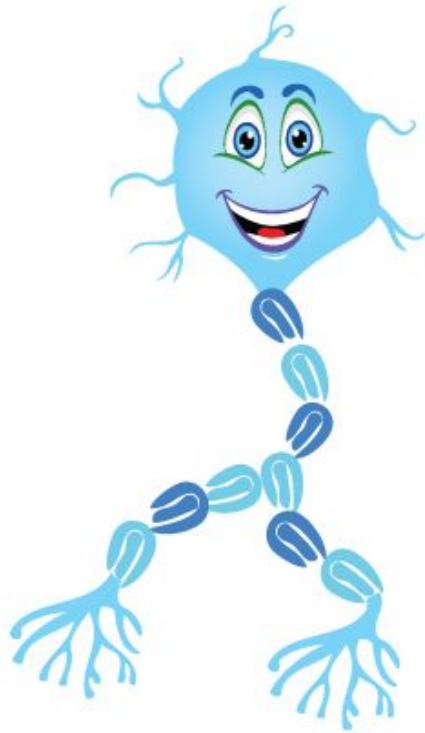
Neural Network: without the brain stuff

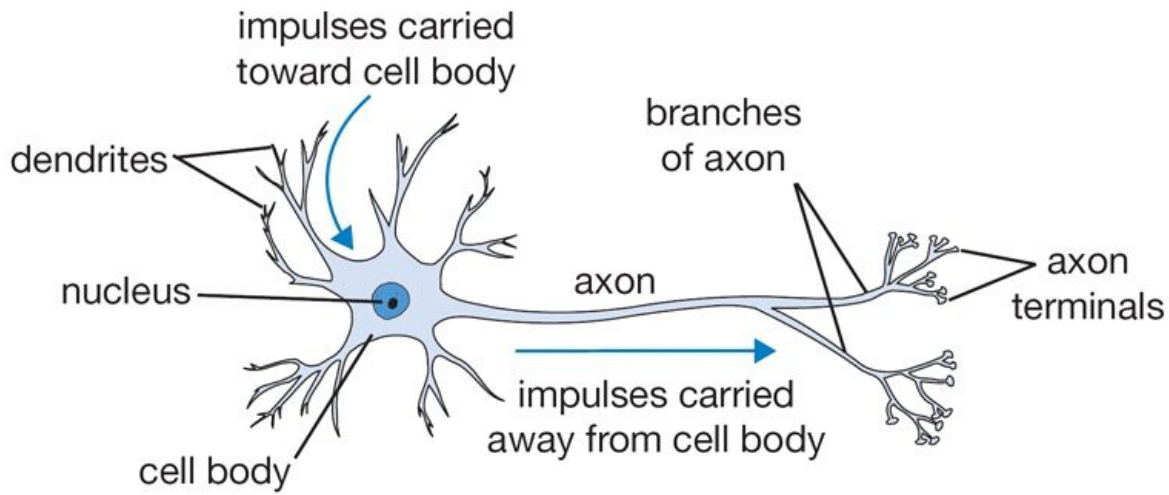
(**Before**) Linear score function: $f = Wx$

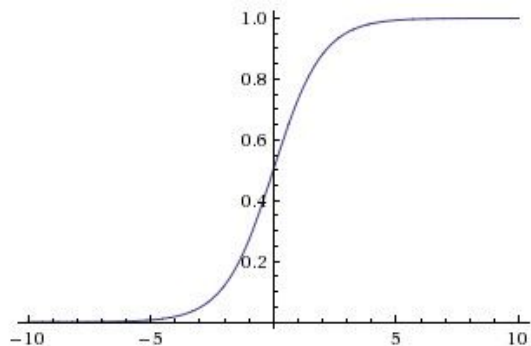
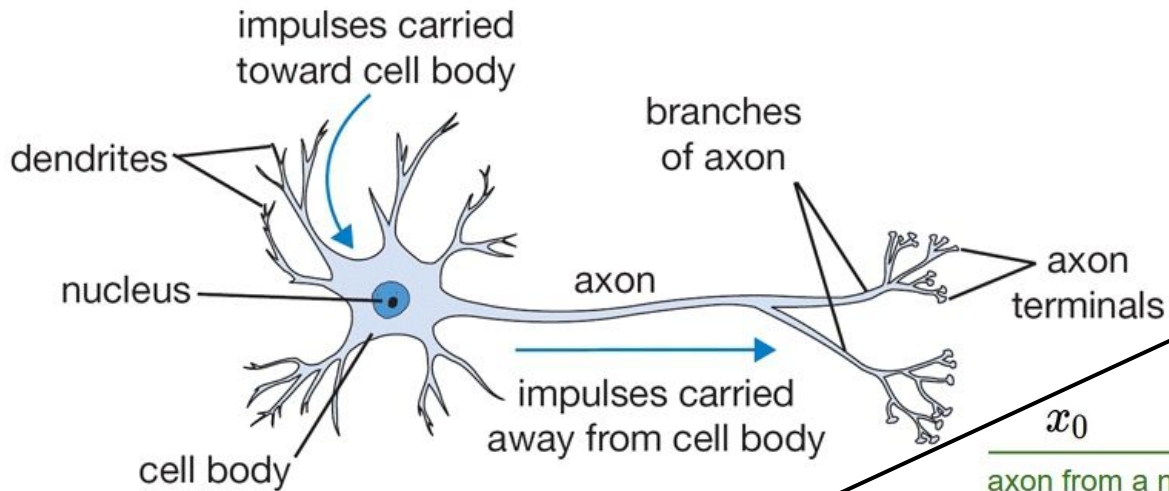
(**Now**) 2-layer Neural Network
or 3-layer Neural Network $f = W_2 \max(0, W_1 x)$

$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$



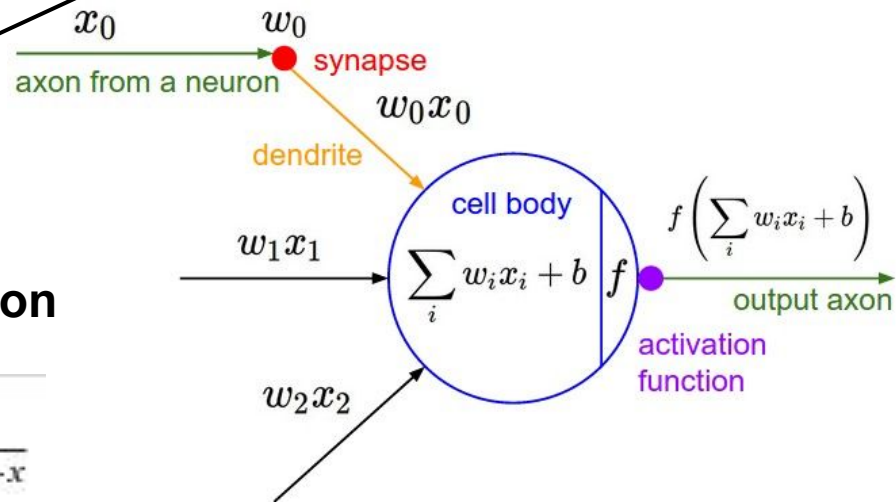






sigmoid activation function

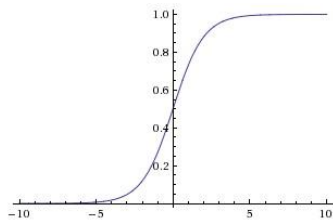
$$\frac{1}{1 + e^{-x}}$$



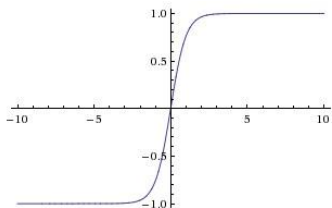
Activation Functions

Sigmoid

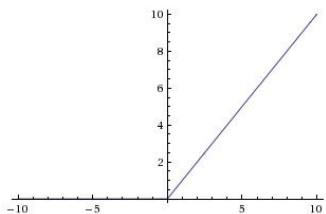
$$\sigma(x) = 1/(1 + e^{-x})$$



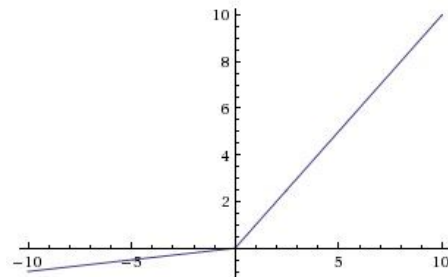
tanh tanh(x)



ReLU max(0,x)



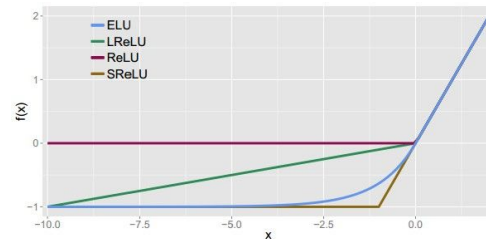
Leaky ReLU max(0.1x, x)



Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

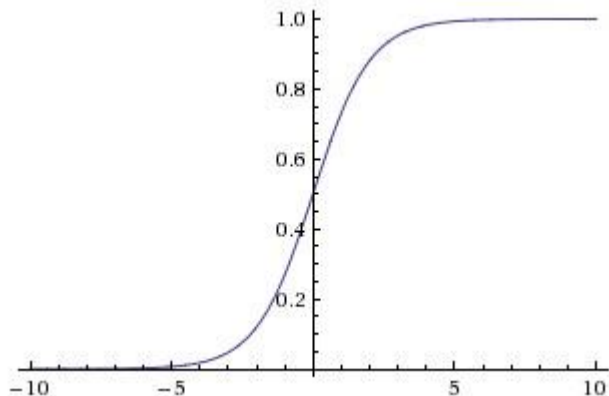
$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$



Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

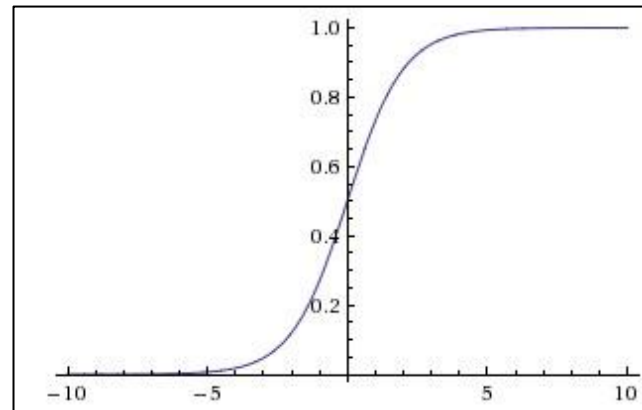
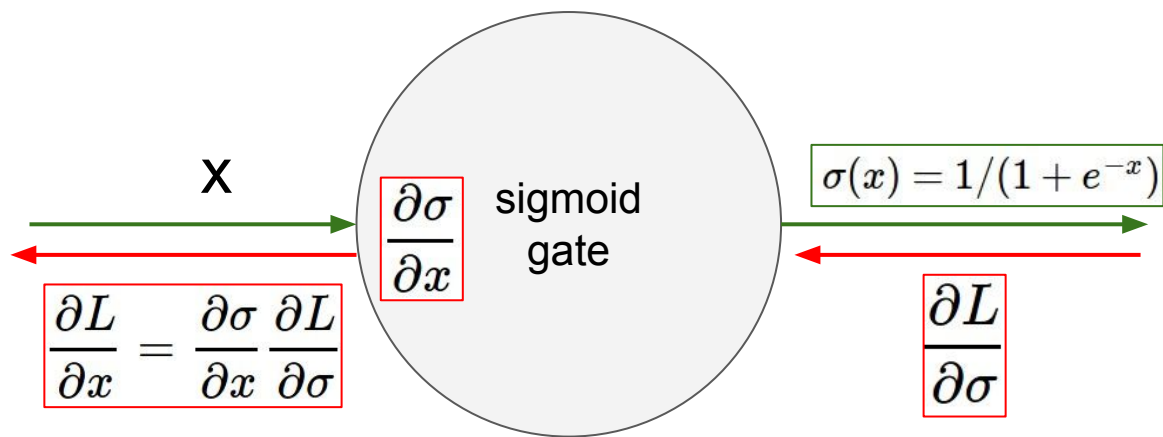
- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron



Sigmoid

3 problems:

1. Saturated neurons “kill” the gradients



What happens when $x = -10$?

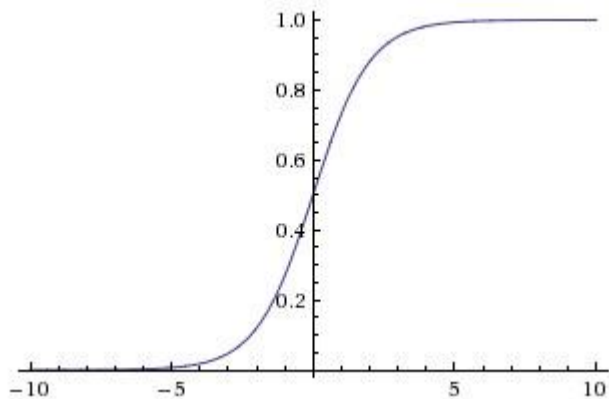
What happens when $x = 0$?

What happens when $x = 10$?

Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

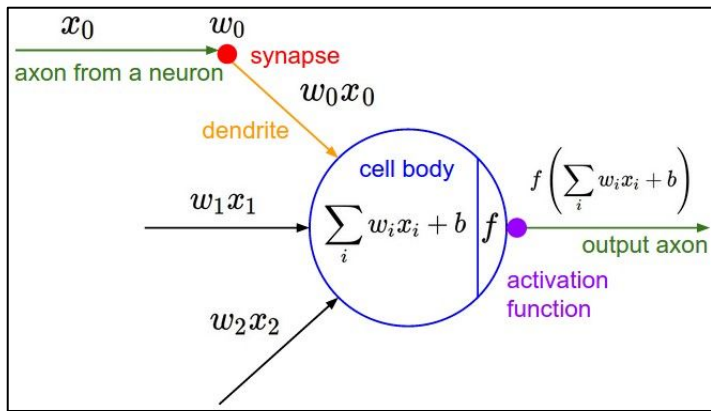


Sigmoid

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron (x) is always positive:

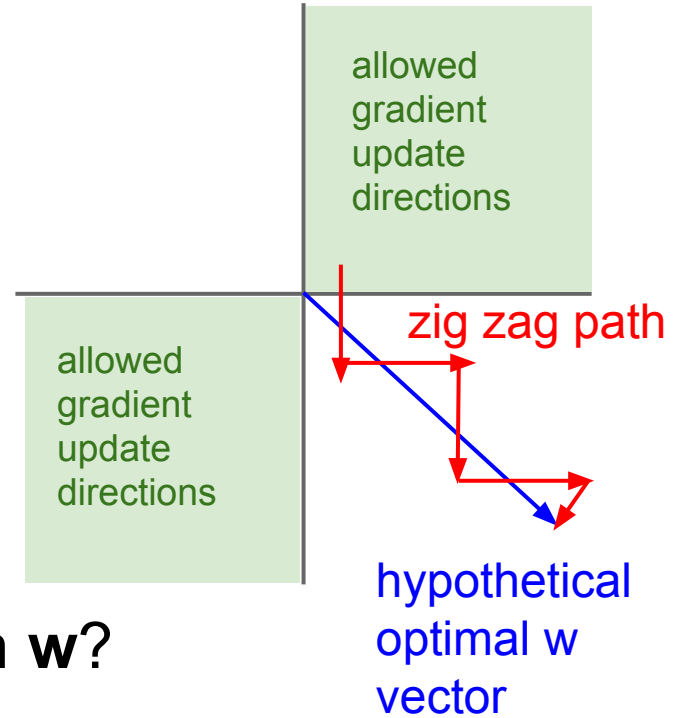


$$f\left(\sum_i w_i x_i + b\right)$$

What can we say about the gradients on \mathbf{w} ?

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$



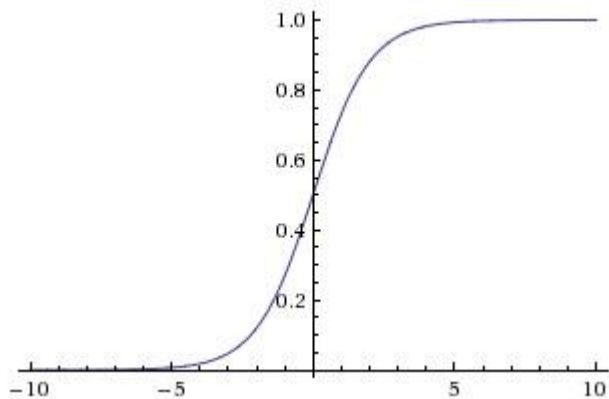
What can we say about the gradients on \mathbf{w} ?

Always all positive or all negative :(
(this is also why you want zero-mean data!)

Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

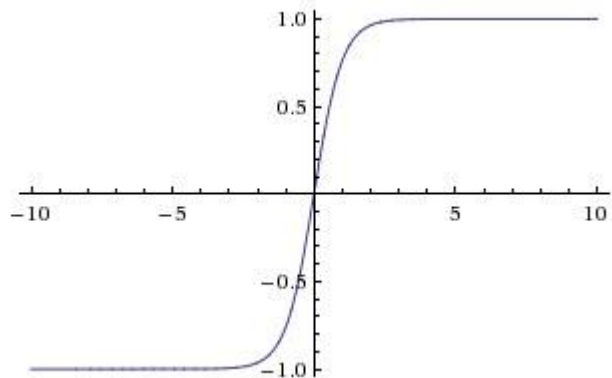


Sigmoid

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered
3. $\exp()$ is a bit compute expensive

Activation Functions



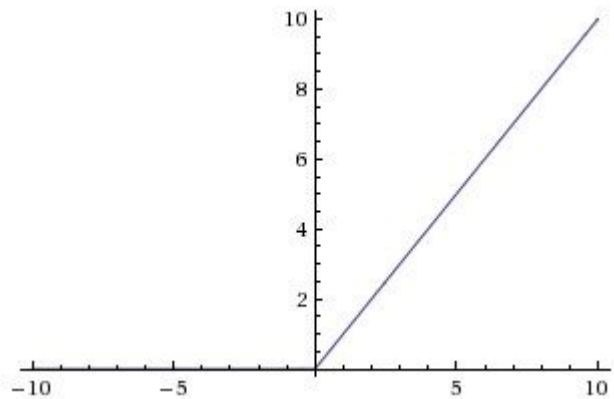
$\tanh(x)$

- Squashes numbers to range $[-1,1]$
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

Activation Functions

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

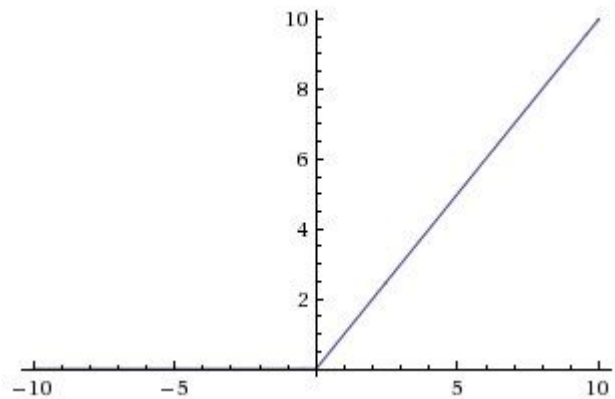


ReLU

(Rectified Linear Unit)

[Krizhevsky et al., 2012]

Activation Functions

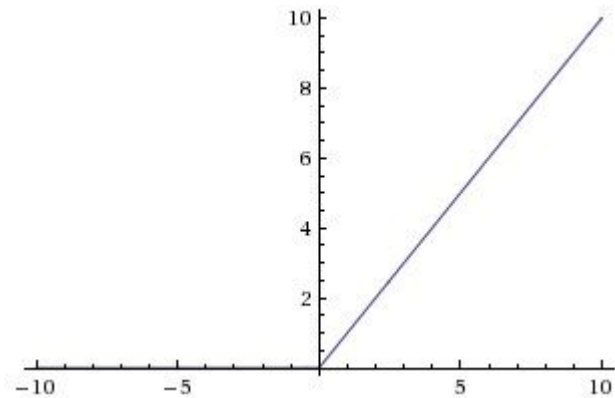
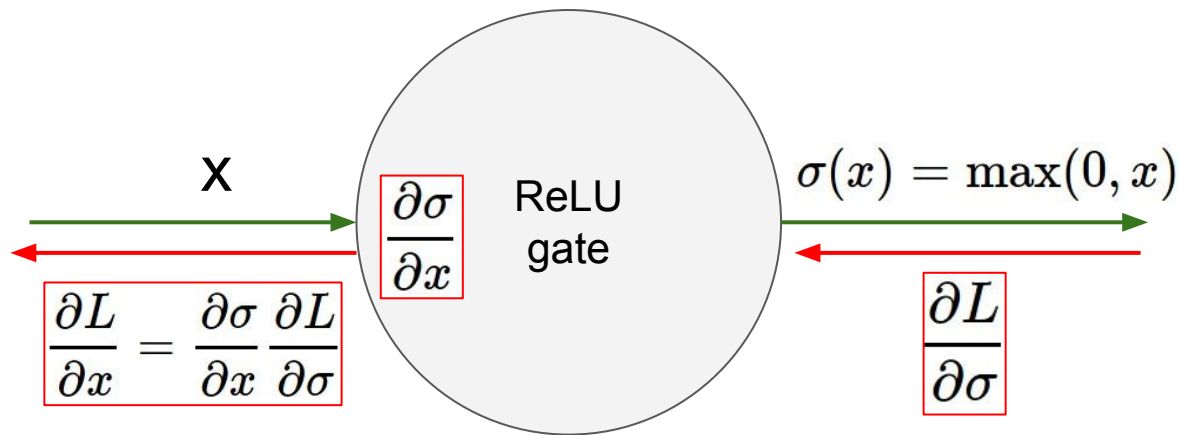


ReLU

(Rectified Linear Unit)

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Not zero-centered output
- An annoyance:

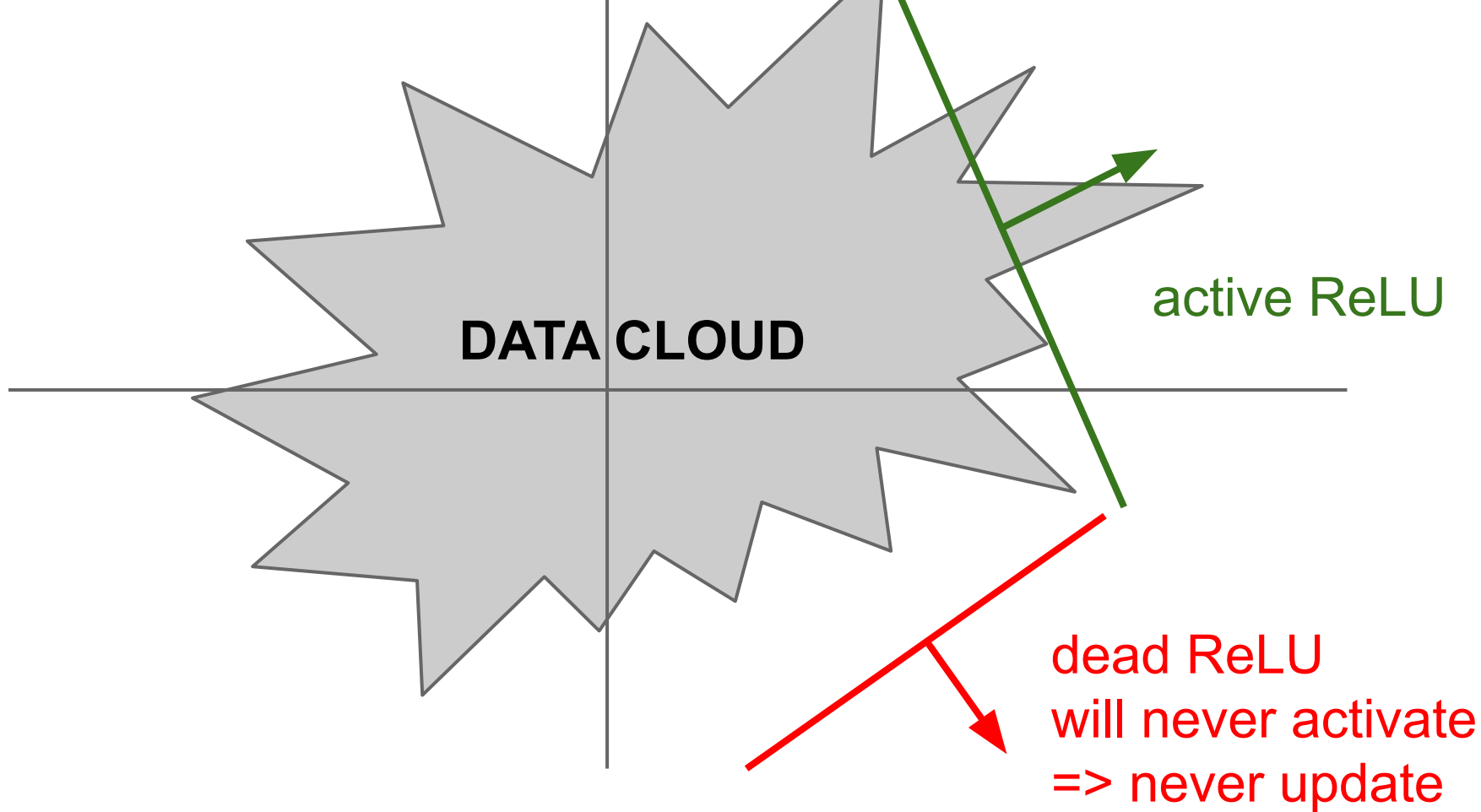
hint: what is the gradient when $x < 0$?

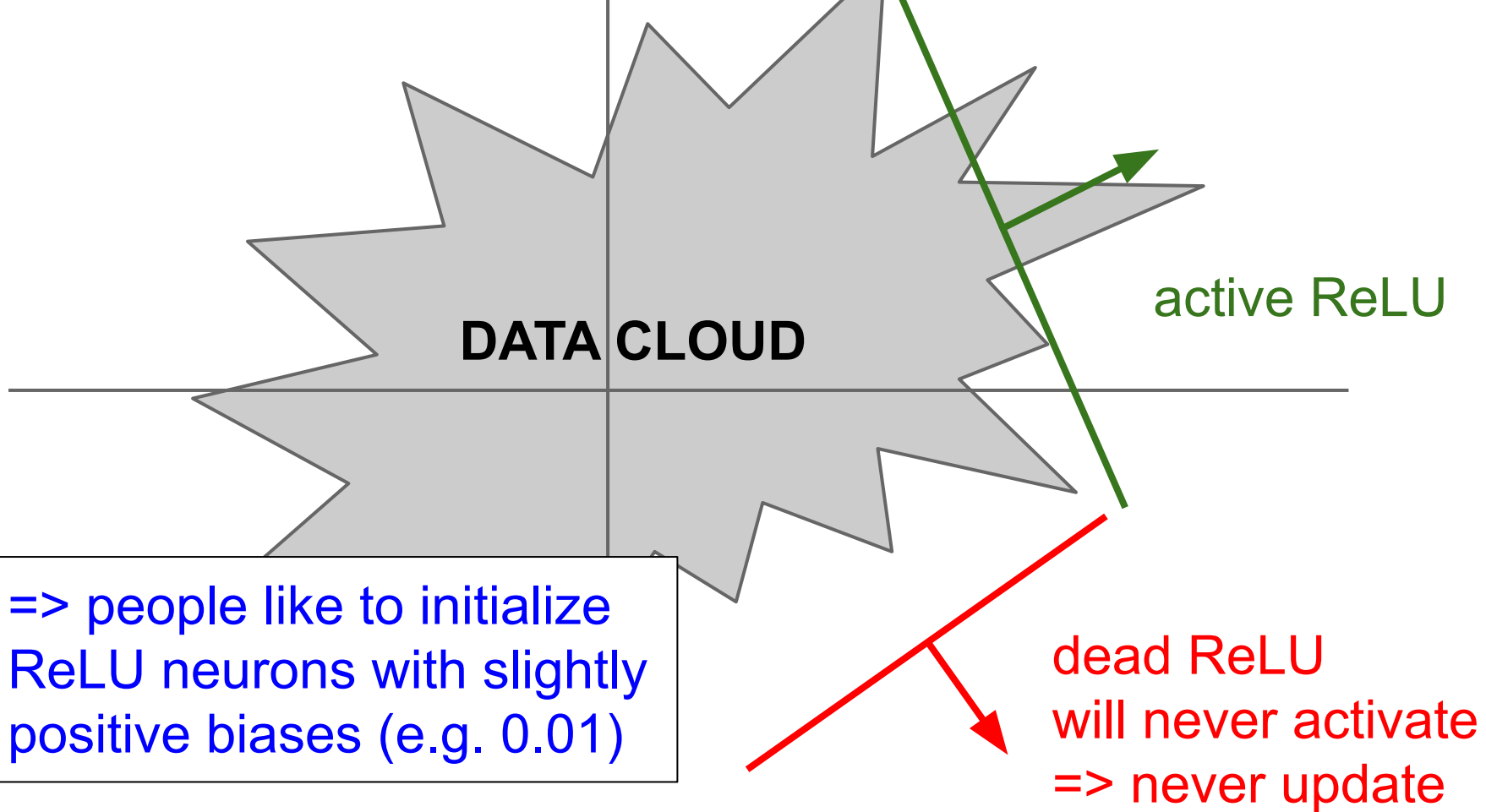


What happens when $x = -10$?

What happens when $x = 0$?

What happens when $x = 10$?

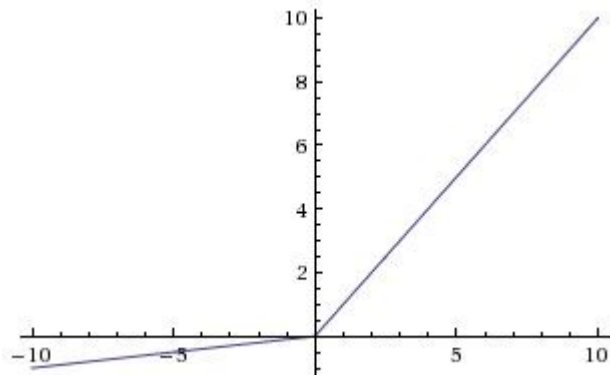




Activation Functions

[Mass et al., 2013]

[He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

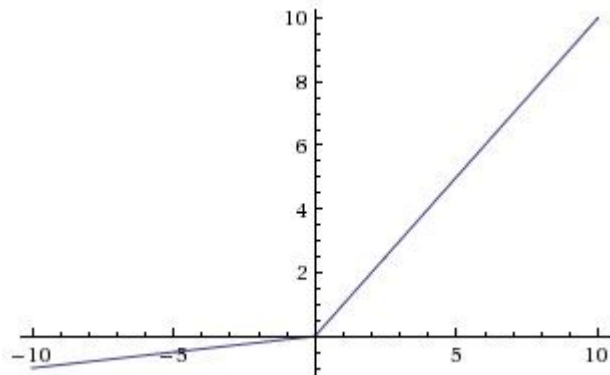
Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Activation Functions

[Mass et al., 2013]

[He et al., 2015]



Leaky ReLU

$$f(x) = \max(0.01x, x)$$

- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”**.

Parametric Rectifier (PReLU)

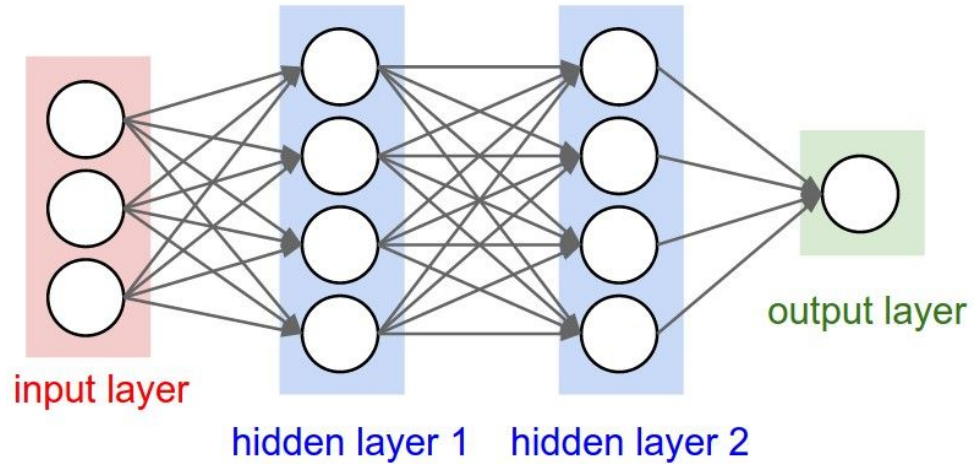
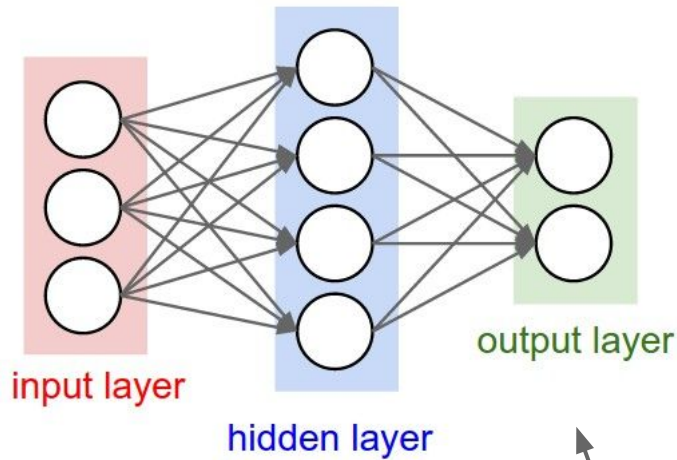
$$f(x) = \max(\alpha x, x)$$

backprop into α
(parameter)

TLDR: In practice:

- Use **ReLU**. Be careful with your learning rates
- Try out **Leaky ReLU / Maxout / ELU**
- Try out **tanh** but don't expect much
- **Don't use sigmoid**

Neural Networks: Architectures

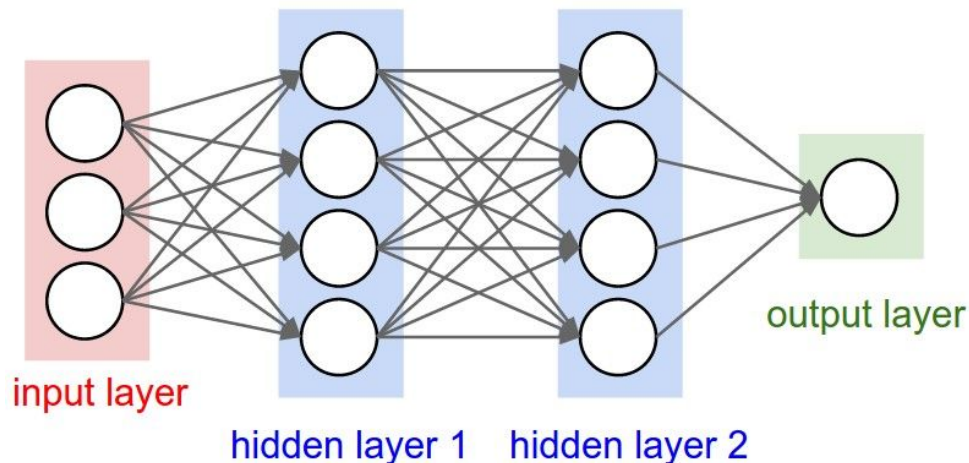


"2-layer Neural Net", or
"1-hidden-layer Neural Net"

"3-layer Neural Net", or
"2-hidden-layer Neural Net"

"Fully-connected" layers

Example Feed-forward computation of a Neural Network



```
# forward-pass of a 3-layer neural network:
```

```
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)
```

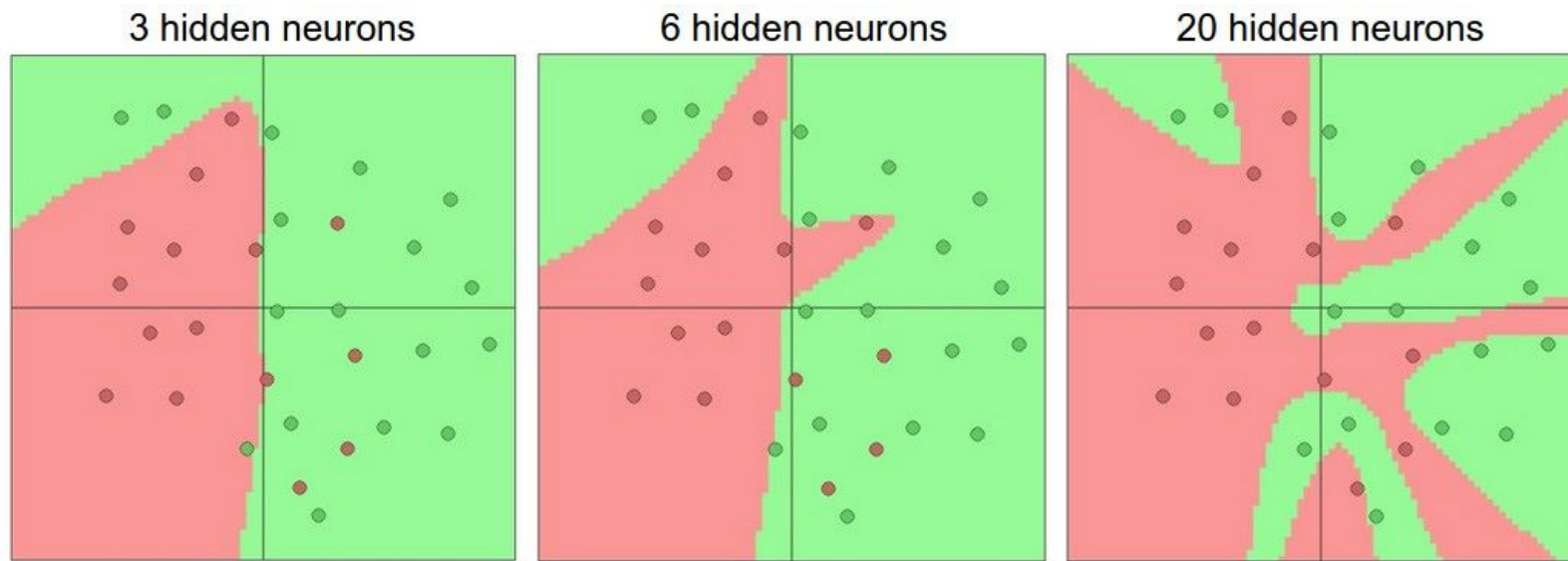
```
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)
```

```
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)
```

```
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)
```

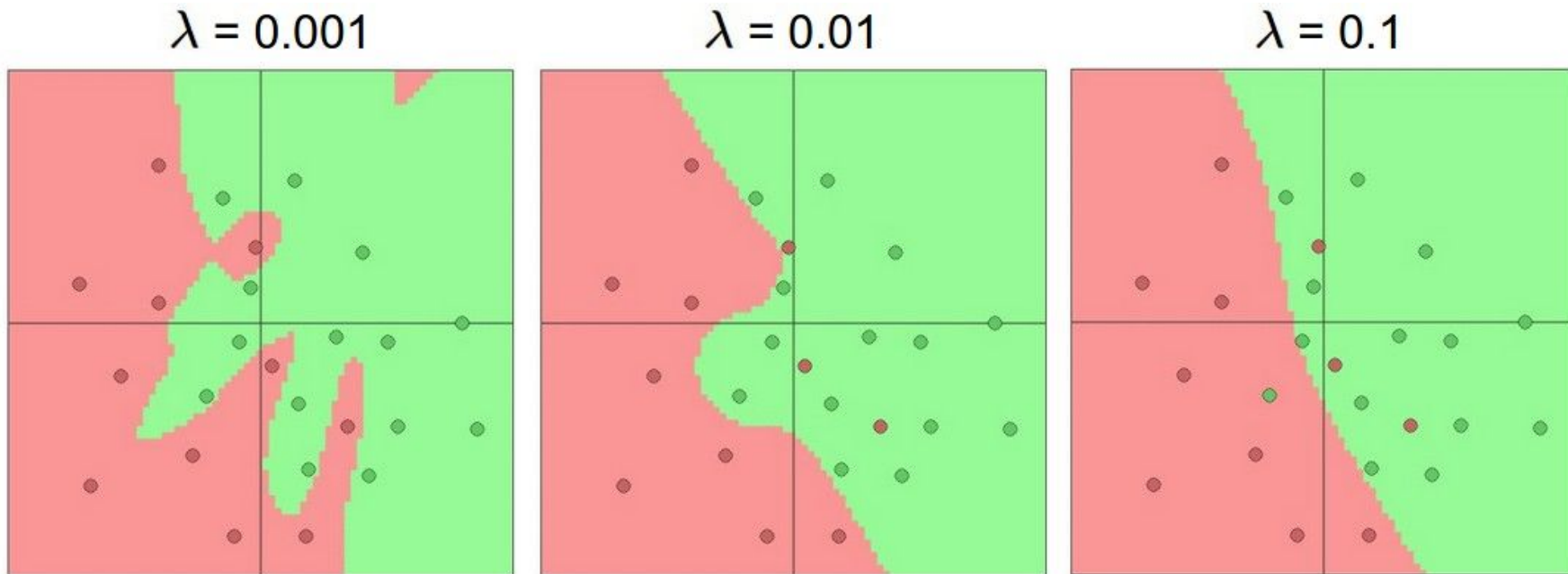
```
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

Setting the number of layers and their sizes



↑
more neurons = more capacity

Do not use size of neural network as a regularizer. Use stronger regularization instead:



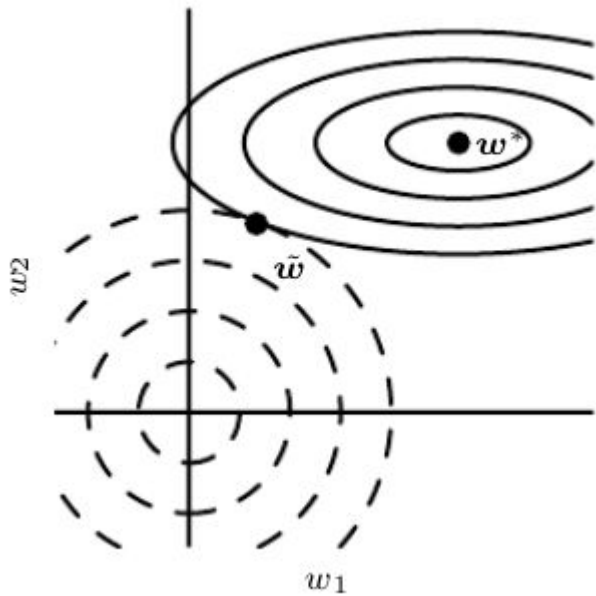
(you can play with this demo over at ConvNetJS:
<http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html>)

L1 regularization on least squares:

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} \sum_j \left(t(\mathbf{x}_j) - \sum_i w_i h_i(\mathbf{x}_j) \right)^2 + \lambda \sum_{i=1}^k |w_i|$$

L2 regularization on least squares:

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} \sum_j \left(t(\mathbf{x}_j) - \sum_i w_i h_i(\mathbf{x}_j) \right)^2 + \lambda \sum_{i=1}^k w_i^2$$



- L1: makes **W** sparse!
- L2: reduces the number of large numbers in **W**!

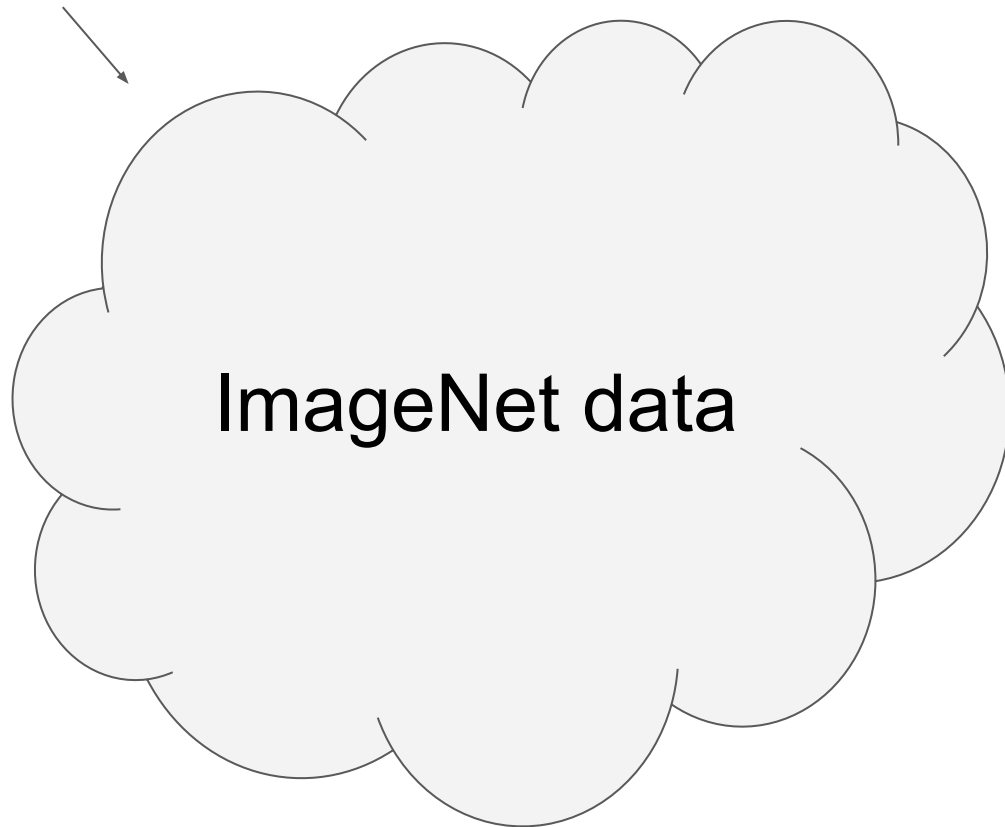
“ConvNets need a lot
of data to train”

“ConvNets need a lot
of data to train”

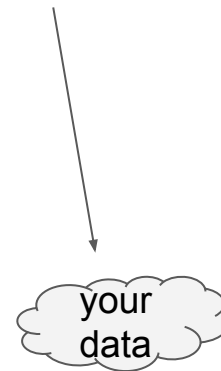


finetuning! we rarely ever
train ConvNets from scratch.

1. Train on ImageNet



2. Finetune network on your own data



Transfer Learning with CNNs



1. Train on ImageNet



2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end



3. If you have medium sized dataset, “**finetune**” instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.